Self-organization in wireless networks: a flow-level perspective

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Abstract

This paper introduces self-optimization for wireless networks taking into account flow-level dynamics. Users arrive and leave the network according to a traffic model. Elastic traffic is considered here. The developed solutions self-optimize the network stability region using user feedback (measurements). The use case considered is cell size optimization. An algorithm is given, and its convergence is proven using stochastic approximation techniques. Convergence points are characterized, allowing performance gains to be evaluated rigorously. Performance gains are evaluated numerically, showing an important increase of the network capacity. [1]

Index Terms

Cellular Networks, Queuing Theory, Stability, Load Balancing, SON, Self Organizing Networks, Stochastic Approximation, Self Optimization

I. Introduction

The concept of self-organization in communication networks, often called Self-Organizing Networks (SON), has been recently recognized as a major axis for the development of future networks by standardization bodies (e.g [1], [2]), network operators, and researchers from both the wired and wireless community. The aim of SON is to introduce autonomic features in networks, such as self-configuration, self-optimization and self-healing (automatic troubleshooting). SON mechanisms would enable the automation of certain tasks performed by network engineers on a daily basis, inducing simplified management, lower operational costs and improved efficiency. Self-optimization involves adapting network parameters to variations of operating conditions such as traffic or propagation. Previous contributions include: Inter-Cell Interference Coordination (ICIC) using a utility-based approach [3], [4] and reinforcement learning [5], and energy savings [6]. Other important SON functionalities are mobility management and load balancing, which is addressed here.

The concept of flow-level dynamics for wireless networks, considering random arrivals and departures of users was introduced in [7]. Works such as [8] or [9] incorporate multi-user diversity gain of channel-aware scheduling in the system model. The flow-level approach is interesting since it introduces the concept of network stability, which is useful for network dimensioning. We say that a network is stable if, without admission control, the number of active users in the network does not grow to infinity. [10] is a well-known example of network optimization to render a network stable whenever that is possible.

The contributions of the present paper are the following:

1) A new approach for self-optimization of wireless networks considering flow-level dynamics is shown, enabling optimization of the network capacity
2) An algorithm for the important SON use-case of cell size optimization (or load balancing) is given
3) Its convergence is proven rigorously using stochastic approximation techniques, its performance is evaluated, and important gains in terms of network capacity are demonstrated

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The paper is organized as follows: Section II describes the system model taking into account flow-level dynamics. In Section III, an algorithm for cell size optimization is stated, and both convergence proofs and performance gain analysis are provided. Section IV concludes the paper. Proofs of all results are found in appendices.

II. Flow-level capacity of wireless networks

We consider a wireless network in a downlink scenario, serving elastic traffic, without user mobility. Users enter the network at random instants and locations, to receive a file of random size. The network area $\mathbb{A} \subset \mathbb{R}^2$ is bounded and convex. Users arrive according to a Poisson process on $\mathbb{A} \times \mathbb{R}$ with measure $\lambda(dr \times dt) = \lambda(r) dr \times dt$, $r \in \mathbb{A}$. This point process models the instants of arrivals and their locations. Users download a file of size σ, with $\mathbb{E}[\sigma] < +\infty$ and we assume independence between the arrival process and the file sizes. Users leave the network when they have finished downloading their file. There are N_{BS} Base Stations (BSs), with $\mathbb{A}_s$ the area served by BS s. We write $R_s(r)$ the data rate of a user located at r served by BS s when there are no other users in s. The stability region of the network is given by Theorem 1 ([7]).

Theorem 1. *Define the load of BS s, ρ_s by:*

$$\rho_s = \mathbb{E}[\sigma] \int_{\mathbb{A}_s} \frac{\lambda(r)}{R_s(r)} dr, \tag{1}$$

and BS s is stable if $\rho_s < 1$, and unstable if $\rho_s > 1$. The network is stable if $\max_s \rho_s < 1$, and unstable if $\max_s \rho_s > 1$.

A station is stable if the probability distribution of the number of active users in this station tends to a stationary distribution. A station is unstable if the number of active users grows to infinity. The total traffic intensity is $\mathbb{E}[\sigma] \int_{\mathbb{A}} \lambda(r) dr$, and the network capacity is the maximal value of the total traffic intensity which ensures stability of the network.

Now assume that when n users are simultaneously served by BS s, the data rate of a user located at r is $\frac{R_s(r) g(n)}{n}$, with $n \to g(n)$ a non-decreasing function and let $g^* = \lim_{n \to +\infty} g(n)$. The function $g(n)$ stands for the multi-user diversity gain which is a characteristic of wireless networks, due to fast-fading. Namely, while there are several transfers in parallel, the base station has access to the channel quality of each user and is able to schedule them when they are in a good fading state. Previous works such as [11], [12], [9] give methods to calculate $g(n)$ in a variety of physical layer settings. Then Theorem 1 applies when replacing ρ_s by $\frac{\rho_s}{g^*}$. See for example ([8]).

III. Cell Size Optimization

We consider the problem of cell size optimization: BSs can adapt the area they serve by adjusting certain parameters. For instance, if a user arriving in the network attaches himself to the BS with the strongest received pilot signal, the BS can choose their transmit pilot power in order to adjust their cell loads. This introduces a form of load balancing, and we show a method to do so based solely on network measurements (user feedback), in a distributed way, and with minimal information exchange between BSs. BSs do not change the power they transmit on data channels, so that the data rates $R_s(r)$ are constant. Only the pilot power can change, and hence the zones $\mathbb{A}_s$ served by the BSs. We define P_s the transmitted pilot power of BS s , and $\mathbf{P} = \{P_s\}_{1 \le s \le N_{BS}}$ - the corresponding vector.

We will show that the cell load can be estimated without bias, and propose a power update mechanism. The convergence of the power update mechanism to an optimal configuration will be demonstrated by studying an associated Ordinary Differential Equation (ODE).

A. Load estimation

Time is divided into slots of size $T > 0$, and the k-th time slot is $[kT, (k+1)T)$. Let $P_s[k]$ be the pilot power transmitted by BS s during the k-th time slot, $\mathbf{P}[k] = \{P_s[k]\}_{1 \le s \le N_{BS}}$ - the corresponding vector, $N_s[k]$ - the number of users that have arrived in $\mathbb{A}_s$ during the k-th time slot and $R_{s,i}[k]$, $\sigma_i[k]$, $1 \le i \le N_s[k]$ - the data rate and file size of the i-th user arriving to BS s during the k-th time slot.

A load estimate for BS s is:

$$\rho_s[k] = \frac{1}{T} \sum_{i=1}^{N_s[k]} \frac{\sigma_i[k]}{R_{s,i}[k]}. \tag{2}$$

The estimate (2) is unbiased and has finite variance, as stated by Theorem 2.

Theorem 2. $\mathbb{E}\left[\rho_s[k]\right] = \rho_s(\mathbf{P}[k])$ *and if* $\mathbb{E}\left[\sigma^2\right] < +\infty$, $\sup_{k \in \mathbb{N}} \mathbb{E}\left[\rho_s[k]^2\right] < +\infty$.

B. Power update mechanism

We propose the following power control mechanism:

$$P_s[k+1] = P_s[k](1 + \epsilon_k(\rho_1[k] - \rho_s[k])). \tag{3}$$

We define the associated ODE:

$$\dot{P}_s = P_s[\rho_1(\mathbf{P}) - \rho_s(\mathbf{P})]. \tag{4}$$

We have chosen BS 1 as the reference station without loss of generality i.e. $P_1[k] = P_1[0]$, $k \in \mathbb{N}$, since the reference station can be changed by permutation of indices.

C. ODE

We now demonstrate several properties of the ODE (4). Let $h_{s,r}$ denote the signal attenuation between BS s and location $r \in \mathbb{A}$, $r_s \in \mathbb{A}$ - the location of BS s, $\mu(\lambda, A) = \int_A \lambda(r)dr$ - with A a Borel set, and $\mu(A) = \int_A dr$.

Assumptions 1. *(i) Users attach themselves to the BS with the strongest received pilot signal:* $\mathbb{A}_s = \{r | s = \arg\max_s h_{s,r} P_s\}$.

(ii) Data rates are upper and lower bounded, with $0 < R_{min} \leq R_s(r) \leq R_{max} < +\infty$, $\forall s, r$. *As a consequence,* $\frac{1}{R_{max}} \leq \frac{\rho_s}{\mu(\lambda,\mathbb{A}_s)\mathbb{E}[\sigma]} \leq \frac{1}{R_{min}}$.

(iii) Signal attenuation is a function of the distance to the BS with $\alpha > 0$, $A > 0$ *and* $h_{s,r} = \frac{A}{\|r_s - r\|_2^\alpha}$.

Theorem 3. *Under Assumptions 1,* $\mathbf{P} \to \rho_s(\mathbf{P})$ *is Lipschitz continuous on* $\mathcal{P} = [P_{min}, +\infty)^{N_{BS}}$, *with* $P_{min} > 0$.

This result will serve to prove unicity of the solutions to the ODE. Another important property is that the maximal load decreases on the trajectories of the ODE, as shown by Theorem 4.

Theorem 4. *Under Assumptions 1:*

(i) Given an initial condition, the ODE (4) *has a unique solution defined on* $\mathbb{R}^+$. *Furthermore it verifies* $0 < \inf_{t \in \mathbb{R}^+} P_s(t) \leq \sup_{t \in \mathbb{R}^+} P_s(t) < +\infty$.

(ii) All solutions of the ODE (4) *converge to a set* $\mathcal{L}$ *on which* $\max_s \rho_s = \min_s \rho_s$.

This theorem has the following consequences: first since the solution is unique given an initial condition, the asymptotic behaviour of the system can be evaluated numerically by standard numerical analysis techniques. Furthermore the transmit power of each station can never be 0 and remains bounded. Finally, the solution converges to a set on which the loads of all stations are equal, namely it performs load balancing. We write $\rho_\infty = \sup_{\mathbf{P} \in \mathcal{L}} \max_s \rho_s(\mathbf{P})$. $\rho_\infty < 1$ implies that the algorithm achieves stability regardless of the initial condition. The algorithm increases the capacity of the network, and the increase in capacity can be computed by evaluating ρ_∞ numerically.

D. Stochastic approximation

Finally we show the link between the asymptotic behaviour of the discrete algorithm (3) and the previously studied ODE (4) through a stochastic approximation result.

Theorem 5. *Assume* $\sum_{k \in \mathbb{N}} \epsilon_k = +\infty$ *and* $\sum_{k \in \mathbb{N}} \epsilon_k^2 < +\infty$ *then* $\max_s \rho_s[k] - \min_s \rho_s[k] \underset{k \to +\infty}{\to} 0$ *and* $\limsup_{k \to +\infty} \max_s \rho_s[k] \leq \rho_\infty$.

E. Numerical experiments

We can now assess the performance gains of the proposed algorithm numerically. The parameters of the network model are given in Table I. We apply a small random perturbation to the base station locations, because in the case of a perfectly hexagonal network, all cells have the same load, and there is no point in trying to perform load balancing. The asymptotic behaviour of the proposed algorithm (Theorem 5) is equivalent to the limit sets of the ODE (4). To evaluate the performance gains numerically, we choose an initial power configuration uniformly distributed in $\mathcal{P}$, and find the corresponding limit set numerically. We repeat the process several times to obtain several limit sets, and for each of them we calculate the network capacity. Figure 1 shows the complementary

Model parameters	
Network layout	Hexagonal
Antenna type	Tri-sector
Number of base stations	16 sites $\times$ 3 sectors
Inter-site distance	$500m$
Network Area	$1km \times 1km$
Access technology	OFDMA
Link Model	SISO, AWGN + Rayleigh fading
Number of resource blocks	12
Resource block size	$180kHz$
BS maximal transmit power	$46dBm$
Thermal noise	$-174dBm/Hz$
Path loss model	$128 + 37.6\log_{10}(d)$ dB, d in km
File size	$10Mbytes$

TABLE I
MODEL PARAMETERS

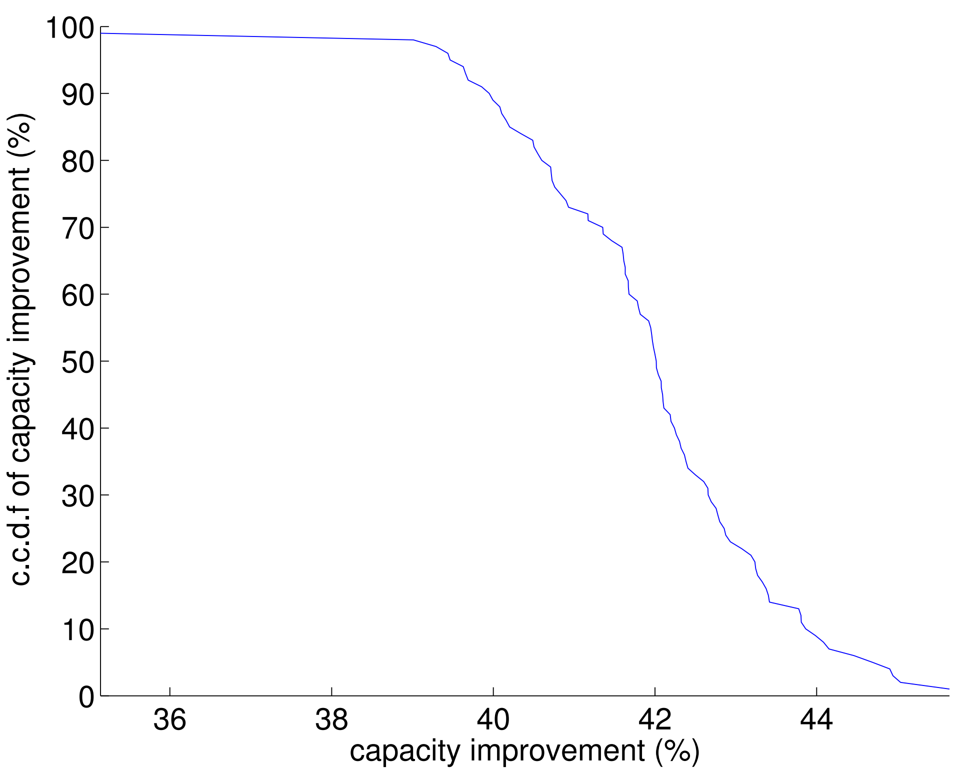

Fig. 1. Cell size optimization: c.c.d.f of performance gains on limit sets of the ODE

cumulative distribution function (c.c.d.f) of the network capacity improvement on the limit sets obtained by the procedure described above. The capacity improvement is calculated with respect to a reference scenario in which

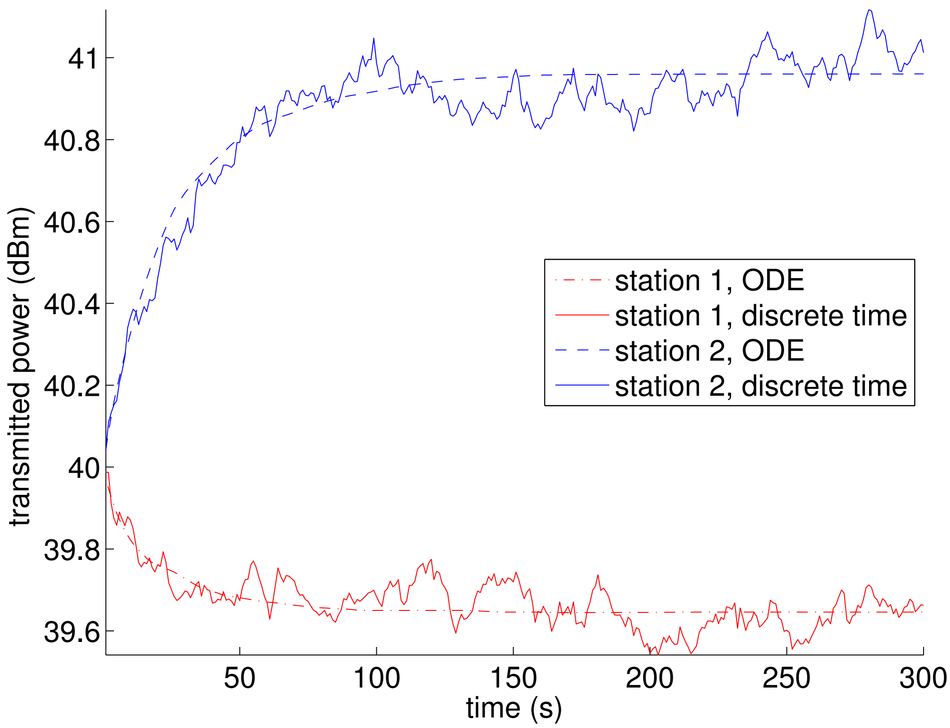

Fig. 2. Cell size optimization: comparison between the discrete time algorithm and the ODE

all base stations transmit the same power. We observe a performance gain of 36% in the worst case and 45% in the best case. The difference between the best and worst case is not very large, suggesting that the proposed method achieves a good performance without a global search. The gain in term of network capacity is considerable. Figure 2 compares the behaviour of the discrete time algorithm obtained by simulating user arrivals with the corresponding trajectory of the ODE. The asymptotic behaviour of the discrete time algorithm is indeed described by the limit set of the ODE.

IV. Conclusion

We have considered the problem of self-optimization in wireless networks, taking into account flow-level dynamics. This approach allows to optimize the stability region of the network, ensuring that the number of active users does not grow to infinity. An algorithm for cell size optimization has been provided, and its convergence has been proven rigorously using stochastic approximation techniques. The performance of the algorithm has been assessed, and important gains in terms of network capacity have been demonstrated. The convergence proof of the self-optimizing algorithm described in this paper is of particular importance to create trust among network operators and to support large scale deployment of SON technology in wireless networks.

Proof of Theorem 2

Proof: Since $\{\frac{\sigma_i[k]}{R_{s,i}[k]}\}_i$ is independent and identically distributed (i.i.d) with $\mathbb{E}\left[\frac{\sigma_i[k]}{R_{s,i}[k]}\right] = \mathbb{E}[\sigma] \frac{\int_{\mathbb{A}_s(\mathbf{P[k]})} \frac{\lambda(r)}{R_s(r)} dr}{\int_{\mathbb{A}_s(\mathbf{P[k]})} \lambda(r) dr}$ and that $\mathbb{E}[N_s[k]] = T \int_{\mathbb{A}_s(\mathbf{P[k]})} \lambda(r) dr < +\infty$, Wald's identity gives that:

$$\mathbb{E}[\rho_s[k]] = \mathbb{E}[\sigma] \int_{\mathbb{A}_s(\mathbf{P[k]})} \frac{\lambda(r)}{R_s(r)} dr = \rho_s(\mathbf{P}[k]). \quad (5)$$

Applying Wald's identity again: $\mathbb{E}\left[\rho_s[k]^2\right] \leq \frac{\mu(\lambda,\mathbb{A}_s)^2 \mathbb{E}[\sigma]^2 + \frac{\mu(\lambda,\mathbb{A}_s)^2}{T} \mathbb{E}[\sigma^2]}{R_{min}^2}$, and $\sup_{k \in \mathbb{N}} \mathbb{E}\left[\rho_s[k]^2\right] < +\infty$ concluding the demonstration. ■

Proof of Theorem 3

Proof: We first consider $\mathbb{A} = [-X_{max}, X_{max}]^2$, and two stations. Station 1 is located at $(-\frac{d}{2}, 0)$, and station 2 at $(\frac{d}{2}, 0)$. By solving the algebraic equation:

$$P_1((x+\frac{d}{2})^2+y^2)^{-\frac{\alpha}{2}} = P_2((x-\frac{d}{2})^2+y^2)^{-\frac{\alpha}{2}}, \tag{6}$$

we have that:

- If $P_1 = P_2$, $\mathbb{A}_1 = \{(x,y)| -X_{max} \leq x \leq 0, -X_{max} \leq y \leq X_{max}\}$
- If $P_1 < P_2$, $\mathbb{A}_1$ is the intersection between $\mathbb{A}$ and a disk of radius $r(P_1,P_2) = d\frac{P_1^{-\frac{1}{\alpha}}P_2^{-\frac{1}{\alpha}}}{|P_1^{-\frac{2}{\alpha}}-P_2^{-\frac{2}{\alpha}}|}$ centered at $(-c(P_1,P_2),0)$ with $c(P_1,P_2) = \frac{d}{2}\frac{P_1^{-\frac{2}{\alpha}}+P_2^{-\frac{2}{\alpha}}}{|P_1^{-\frac{2}{\alpha}}-P_2^{-\frac{2}{\alpha}}|}$
- If $P_1 > P_2$, $\mathbb{A}_2$ is the intersection between $\mathbb{A}$ and a disk of radius $r(P_1,P_2)$ centered at $(c(P_1,P_2),0)$.

Assume that $P_1 < P_2$:

$$\mu(\mathbb{A}_1(P_1,P_2)) = \int_0^{\pi} R(\theta,P_1,P_2)^2 d\theta, \tag{7}$$

with:

$$\begin{aligned} R(\theta,P_1,P_2)^2 &= \min(r(P_1,P_2)\sin(\theta), X_{max})^2 \\ &+ \max(r(P_1,P_2)\cos(\theta), c(P_1,P_2)-X_{max})^2. \end{aligned} \tag{8}$$

Since both $(P_1,P_2) \to r(P_1,P_2)$ and $(P_1,P_2) \to c(P_1,P_2)$ are bounded with bounded derivatives in a neighborhood of (P_1,P_2), $(P_1,P_2) \to \mu(\mathbb{A}_1(P_1,P_2))$ is locally Lipschitz continuous at (P_1,P_2). By symmetry, the same is true for $P_1 > P_2$.

Now assume that $P_2 > P_1 > 0$ and $|P_2 - P_1| \leq \epsilon$, then there exists $K_4 > 0$ such that:

$$|\mu(\mathbb{A}_1(P_1,P_2)) - \mu(\mathbb{A}_1(P_1,P_1))| \leq K_4\frac{\epsilon}{P_1} + o(\epsilon), \tag{9}$$

hence Lipschitz continuity is valid on $\mathcal{P}$.

We now consider the general case with an arbitrary number of stations. We consider N_{BS} stations, $\mathbf{P^{(1)}} \in \mathcal{P}$, $\mathbf{P^{(2)}} \in \mathcal{P}$, and without loss of generality $\rho_s(\mathbf{P^{(1)}}) \geq \rho_{\mathbf{s}}(\mathbf{P^{(2)}})$. Let $\mathbf{P^{(3)}} \in \mathcal{P}$,$\mathbf{P^{(4)}} \in \mathcal{P}$ with $P_{s'}^{(3)} = \min(P_{s'}^{(1)}, P_{s'}^{(2)})$, $s' \neq s$ and $P_s^{(3)} = \max(P_s^{(1)}, P_s^{(2)})$, $P_{s'}^{(4)} = \max(P_{s'}^{(1)}, P_{s'}^{(2)})$ and $P_s^{(4)} = \min(P_s^{(1)}, P_s^{(2)})$. It is noted that $\|\mathbf{P^{(2)}} - \mathbf{P^{(1)}}\|_\infty = \|\mathbf{P^{(3)}} - \mathbf{P^{(4)}}\|_\infty$.

Since: $|\rho_s(\mathbf{P^{(1)}}) - \rho_{\mathbf{s}}(\mathbf{P^{(2)}})| \leq |\rho_{\mathbf{s}}(\mathbf{P^{(3)}}) - \rho_s(\mathbf{P^{(4)}})|$ and $\mathbb{A}_s(\mathbf{P^{(4)}}) \subset \mathbb{A}_s(\mathbf{P^{(3)}})$, $|\rho_s(\mathbf{P^{(3)}}) - \rho_s(\mathbf{P^{(4)}})| \leq K_2\mu(\mathbb{A}_s(\mathbf{P^{(3)}}) \backslash \mathbb{A}_{\mathbf{s}}(\mathbf{P^{(4)}}))$. We write: $\mathbb{A}_{s,s'}(\mathbf{P}) = \{r | h_{s,r}P_s \geq h_{s',r}P_{s'}\}$, and $\mathbb{A}_s(\mathbf{P}) = \cap_{s' \neq s}\mathbb{A}_{s,s'}(\mathbf{P})$. Furthermore:

$$\mathbb{A}_s(\mathbf{P^{(3)}}) \setminus \mathbb{A}_s(\mathbf{P^{(4)}}) \subset \cup_{s' \neq s}(\mathbb{A}_{s,s'}(\mathbf{P^{(3)}}) \setminus \mathbb{A}_{s,s'}(\mathbf{P^{(4)}})). \tag{10}$$

Hence we have that:

$$|\rho_s(\mathbf{P^{(1)}}) - \rho_{\mathbf{s}}(\mathbf{P^{(2)}})| \leq \sum_{\mathbf{s'} \neq \mathbf{s}} \mu(\mathbb{A}_{\mathbf{s},\mathbf{s'}}(\mathbf{P^{(3)}}) \setminus \mathbb{A}_{\mathbf{s},\mathbf{s'}}(\mathbf{P^{(4)}})), \tag{11}$$

which proves the result, since there exists K_3 so that $\mu(\mathbb{A}_{s,s'}(\mathbf{P^{(3)}}) \setminus \mathbb{A}_{s,s'}(\mathbf{P^{(4)}})) \leq K_3\|\mathbf{P^{(3)}} - \mathbf{P^{(4)}}\|_2$ by using the result obtained for two stations. ■

Lemma 1

Lemma 1. *Let $x : \mathbb{R} \to \mathbb{R}^n$, absolutely continuous with almost everywhere (a.e) derivative $\dot{x}(t)$. Then $t \to \min_s x_s(t)$ and $t \to \max_s x_s(t)$ are absolutely continuous, with derivatives $\dot{x}_{s(t)}(t)$, $s(t) \in \{\arg\min_s x_s(t)\}$, and $\dot{x}_{s(t)}(t)$, $s(t) \in \{\arg\max_s x_s(t)\}$.*

Proof: $t \to \min_s x_s(t)$ and $t \to \max_s x_s(t)$ are absolutely continuous by composition of an absolutely continuous function and a Lipschitz continuous function.

We write W the set of points where at least one component of $x(t)$ is not-differentiable, and $\mu(W) = 0$. We consider $t \in [t_0, t_1] \setminus W$.

We first assume that for all $(s_1, s_2) \in \{\arg\max_s x_s(t)\}$ we have $\dot{x}_{s_1}(t) = \dot{x}_{s_2}(t)$. A Taylor development gives that $t \to \max_s x_s(t)$ is differentiable at t with derivative $\dot{x}_s(t)$, with $s \in \{\arg\max_s x_s(t)\}$.

We now consider the set $U = \{t \in [t_0, t_1] \setminus W : \exists ((s_1, s_2) \in \{\arg\max_s x_s(t)\}, \dot{x}_{s_1}(t) \neq \dot{x}_{s_2}(t))\}$. We want to prove that $\mu(U) = 0$.

Let $V(s_1, s_2) = \{t \in [t_0, t_1] \setminus W : x_{s_1}(t) = x_{s_2}(t), \dot{x}_{s_1}(t) \neq \dot{x}_{s_2}(t)\}$. Then all points of $V(s_1, s_2)$ are isolated, hence $\mu(V(s_1, s_2)) = 0$. $U \subset \cup_{s_1 \neq s_2} V(s_1, s_2)$, so $\mu(U) = 0$ by countable union.

So we have proven that $t \to \max_s x_s(t)$ has derivative $\dot{x}_{s(t)}(t)$, $s(t) \in \{\arg\max_s x_s(t)\}$ a.e. The proof for $t \to \min_i x_i(t)$ is straightforward from the proof above. ■

PROOF OF THEOREM 4

Proof: (i) Since $\min_s P_s(0) > 0$, Theorem 3 states that the cell loads are Lipschitz continuous in a neighbourhood of $\mathbf{P}(0)$. Hence $\mathbf{P} \to (\rho_1(\mathbf{P}) - \rho_s(\mathbf{P}))$ is Lipschitz continuous in a neighbourhood of $\mathbf{P}(0)$, and the Picard-Lindelöf theorem ensures that there exists an unique local solution given an initial condition in $\mathcal{P}$.

Upper bound Consider such a local solution defined on $[0, \delta)$, $t \in [0, \delta)$, and assume $P_s(t) = \max_s P_s(t) > P_{max}$, then: $\rho_1(\mathbf{P}(t)) \leq \rho_1(P_1(0), 0, \cdots, 0, P_{max}, 0, \cdots, 0)$ and $\rho_s(\mathbf{P}(t)) \geq \rho_s(P_{max}, \cdots, P_{max}) = \rho_s(1, \cdots, 1)$. Hence since $\rho_1(P_1(0), 0, \cdots, 0, P_{max}, 0, \cdots, 0) \underset{P_{max} \to +\infty}{\to} 0$ and $\rho_s(1, \cdots, 1) > 0$, there exists a value of P_{max} such that if $P_s(t) = \max_s P_s(t) > P_{max}$ then $\dot{P}_s(t) \leq 0$.

Now assume that there exists t_1 such that $\max_s P_s(t) > P_{max}$, there also exists t_0 such that $\max_s P_s(t) = P_{max}$ and $\max_s P_s(t) > P_{max}$, $t \in [t_0, t_1]$. Applying Lemma 1 we obtain $P_{max} < \max_s P_s(t_1) \leq P_{max}$ which is impossible. Hence $\sup_{t \in [0,\delta)} P_s(t) < +\infty$.

Lower bound We write $P_{max} = \sup_{t \in [0,\delta)} \max_s P_s(t)$. Assume that $P_s(t) = \min_s P_s(t) < P_{min}$, then $\rho_1(\mathbf{P}(t)) \geq \rho_1(P_1(0), P_{max}, \cdots, P_{max})$ and $\rho_s(\mathbf{P}(t)) \leq \rho_s(P_1(0), P_{min}, \cdots, P_{min})$. Since $\rho_s(P_1(0), P_{min}, \cdots, P_{min}) \underset{P_{min} \to +\infty}{\to} 0$, there exists a value of P_{min} such that if $P_s(t) = \min_s P_s(t) < P_{min}$ then $\dot{P}_s(t) \geq 0$. Using Lemma 1 and the same argument as above, we obtain that $\inf_{t \in [0,\delta)} P_s(t) > 0$.

Maximality Since $0 < \inf_{t \in [0,\delta)} P_s(t) \leq \sup_{t \in [0,\delta)} P_s(t) < +\infty$, and assuming that $\delta < +\infty$ the considered local solution can be extended to $[0, \delta')$ with $\delta < \delta'$. This proves that the ODE has a unique solution defined on $\mathbb{R}^+$ and that $0 < \inf_{t \in \mathbb{R}^+} P_s(t) \leq \sup_{t \in \mathbb{R}^+} P_s(t) < +\infty$.

(ii) Since $t \to \mathbf{P}(t)$ is absolutely continuous, and $\mathbf{P} \to \rho_s(\mathbf{P})$ is Lipschitz continuous, $t \to \rho_s(\mathbf{P}(t))$ is absolutely continuous and has a derivative a.e, and we write Z the set on which the function is non-differentiable.

Let $t_0 \notin Z$, and $s \in \{\arg\max_s \rho_s(\mathbf{P}(t_0))\}$,

$$\frac{d}{dt} \frac{P_{s'}(t_0)}{P_s(t_0)} = \frac{P_{s'}(t_0)}{P_s(t_0)} [\rho_s(\mathbf{P}(t_0)) - \rho_{s'}(\mathbf{P}(t_0))] \geq 0, \tag{12}$$

with equality if $s' \in \{\arg\max_s \rho_s(\mathbf{P}(t_0))\}$.

Using homogeneity of $\mathbf{P} \to \rho_s(\mathbf{P})$: $\rho_s(\mathbf{P}(t_0 + \epsilon)) = \rho_s\left(\frac{\mathbf{P}(t_0+\epsilon)}{P_s(t_0+\epsilon)}\right)$. Using Lipschitz continuity of ρ_s:

$$\begin{aligned} &\rho_s\left(\frac{\mathbf{P}(t_0+\epsilon)}{P_s(t_0+\epsilon)}\right) \\ &= \rho_s\left(\frac{\mathbf{P}(t_0)}{P_s(t_0)}(1 + \epsilon[\rho_s(\mathbf{P}(t_0)) - \rho(\mathbf{P}(t_0))])\right) + o(\epsilon) \\ &\leq \rho_s\left(\frac{\mathbf{P}(t_0)}{P_s(t_0)}\right) + o(\epsilon) = \rho_s(\mathbf{P}(t_0)) + o(\epsilon) \end{aligned} \tag{13}$$

Hence :

$$\lim_{\epsilon \to 0} \frac{\rho_s(\mathbf{P}(t_0+\epsilon)) - \rho_s(\mathbf{P}(t_0))}{\epsilon} \leq 0 \tag{14}$$

It is noted that the limit exists because of differentiability at t_0. Assume $\max_s \rho_s(t_0) > \min_s \rho_s(t_0)$. If $\{\arg\max_s \rho_s(t_0)\} = \{s_0\}$ there exists $t_1 > t_0$ such that $t \to \max_s \rho_s(t) = \rho_{s_0}(t)$ is strictly decreasing on $[t_0, t_1]$, since $t \to \frac{P_{s_0}(t)}{P_s(t)}$ is strictly decreasing on $[t_0, t_1]$ for $s \neq s_0$.

If $|\{\arg\max_s \rho_s(t_0)\}| > 1$, either $\rho_{s_1}(t) = \rho_{s_2}(t)$ on $[t_0, t_1]$ or there exists $t_0 \leq t_2 < t_1$ such that $|\{\arg\max_s \rho_s(t_2)\}| < |\{\arg\max_s \rho_s(t_0)\}|$. In the first case we must have that $t \to \max_s \rho_s(t)$ is strictly decreasing on $[t_0, t_1]$, since $t \to \frac{P_{s_1}(t)}{P_s(t)}$, $s_1 \in \{\arg\max_s \rho_s(t_0)\}$, $s \notin \{\arg\max_s \rho_s(t_0)\}$ is strictly decreasing on $[t_0, t_1]$. The second case reduces to $|\{\arg\max_s \rho_s(t_0)\}| = 1$ by recurrence on $|\{\arg\max_s \rho_s(t_0)\}|$. We have proven that $t \to \max_s \rho_s(t)$ is strictly decreasing whenever $\max_s \rho_s(t) > \min_s \rho_s(t)$ which concludes the demonstration.

■

PROOF OF THEOREM 5

Proof: Theorem 2 states that: $\mathbb{E}\left[\rho_s[k]\right] = \rho_s(\mathbf{P}[k])$ and $\sup_k \mathbb{E}\left[\rho_s[k]^2\right] < +\infty$. Hence [13][Theorem 2.1, page 127] gives that the sequence $\{\rho[k]\}$ converges almost surely (a.s) to a limit set of the ODE (4). Furthermore we have proven that on each limit set of the ODE, $\min_s \rho_s = \max_s \rho_s \leq \rho_\infty$, which proves the result. ■

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