A Self-Optimization Method for Coverage-Capacity Optimization in OFDMA networks with MIMO

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Outline

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Wireless networks feature channel-aware schedulers, and there is a need to know the complete channel statistics for the evaluation of their performance.

- Multiple antennas techniques (MIMO) allow to boost the network capacity dramatically.
- Self-organisation and Self-optimization (SON) have become important research topics.
- Example of application of SON: automatic outage detection and compensation.
Related work

- Concept of $\alpha$-fairness ($^1$)
- Capacity of MIMO systems ($^2$)
- Asymptotic distribution of the capacity of MIMO systems ($^3$)

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We consider a cell with $N$ users.

The total available bandwidth $W$ is divided in $K$ Resource Blocks (RBs).

Time is slotted, with a set of scheduling instants $(t_m)_{m \in \mathbb{N}}$.

$r_{i,t_m}^{(k)}$ denotes the instantaneous data rate of user $i$ at time $t_m$ on RB $k$.

The mean data rate is calculated by a low-pass filter, with $\epsilon > 0$:

$$
\bar{r}_{i,t_{m+1}}^{(k)} = (1 - \epsilon)\bar{r}_{i,t_{m+1}}^{(k)} + \epsilon \delta_{m+1,i,k} r_{i,t_{m+1}}^{(k)}
$$

(1)

$$
\bar{r}_{i,t_{m+1}} = \sum_{k=1}^{K} \bar{r}_{i,t_{m+1}}^{(k)}
$$

(2)

with $\delta_{m,i,k} = 1$ if user $i$ was allowed to transmit on RB $k$ at $t_m$ and 0 otherwise.
Alpha-fair scheduling: definition

Definition

Given \( d > 0 \), the \( \alpha \)-fair scheduler is the scheduling strategy which maximizes:

\[
U = \begin{cases} 
\sum_{i=1}^{N} \log(d + \bar{r}_i), & \alpha = 1 \\
\sum_{i=1}^{N} \frac{(\bar{r}_i + d)^{1-\alpha} - 1}{1 - \alpha}, & \alpha \neq 1
\end{cases}
\]  

(3)
Alpha-fair scheduling: scheduling rule

**Theorem**

*When* $\epsilon \to 0^+$ *the allocation rule of the* $\alpha$-fair scheduler *is:*

$$
\arg \max_{1 \leq i \leq N} \frac{r_{i,t_{m+1}}^{(k)}}{\left(\bar{r}_{i,t_m} + d\right)^\alpha}
$$

**(4)**

**Sketch of proof:**

- Applying stochastic approximation results, the behaviour of the system as $\epsilon \to 0^+$ reduces to the ODE
  $$\theta(t) = h(\theta(t)) - \theta(t)$$
- The ODE has a solution using the Picard theorem, and we can use the Kamke condition to prove its convergence to an unique $\theta^*$
- We show that $\theta^*$ is a local maximum of $U$, and we conclude using the convexity of $U$
We assume $n_t$ transmit antennas and $n_r$ receive antennas.

- $I_{n_r}$ - the $n_r \times n_r$ identity matrix, $H$ - the $n_r \times n_t$ channel matrix, Rayleigh fading, no antenna correlation.

Given $H$, the instantaneous capacity on RB $k$ is:

$$C_i^{(k)} = \log_2 \left[ \det \left( I_{n_r} + \frac{S_i^{(k)}}{n_t} HH^* \right) \right]$$  \hspace{1cm} (5)

Kamath and Hughes have shown that as $n_{min} = \min(n_t, n_r) \to +\infty$, $C_i^{(k)}$ converges in distribution to a normally distributed random variable with known mean and variance.

The approximation is very accurate even for small values of $n_{min}$. 
Scheduling Gain

- Using the Gaussian approximation, the scheduling gain can be calculated for $\alpha \in \{0, 1, +\infty\}$

  - $\alpha = 0$, Max Throughput

    $$\bar{r}_i = \frac{K}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \left( z\sigma_i + \mu_i \right) \left[ \prod_{j \neq i} F \left( \frac{\mu_j - \mu_i + z\sigma_i}{\sigma_j} \right) \right] e^{-\frac{z^2}{2}} \, dz$$

    (6)

  - $\alpha = 1$, Proportional Fair

    $$\bar{r}_i = \frac{K}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \left( z\sigma_i + \mu_i \right) \left[ \prod_{j \neq i} F \left( z \frac{\mu_i\sigma_j}{\mu_j\sigma_i} \right) \right] e^{-\frac{z^2}{2}} \, dz$$

    (7)

  - $\alpha = +\infty$, Max-min fair

    $$\bar{r}_i = \frac{K}{\sum_{i=1}^{N} \frac{1}{\mu_i}}$$

    (8)
We consider a service with minimal data rate $R_{\text{min}}$, and we want to adjust $\alpha$ dynamically to serve the maximal number of users.

Previous formulas allow to calculate which users can be served, providing that $\alpha$ is large enough.

Every 1s or so, the base station increases $\alpha$ if some users are below $R_{\text{min}}$, and decreases $\alpha$ otherwise.
We consider a multi-cell network where users arrive according to a Poisson process to receive a service with minimal data rate (e.g. streaming) and leave upon completion.

$\alpha$ is adjusted according to the mechanism described above.

Users are dropped if they do not receive the minimal data rate for a certain period of time.

We show that the mechanism decreases the dropping rate appreciably.
Simulation results

Figure: Evolution of $\alpha$ as a function of time for a base station.

Figure: Number of users in a base station as a function of time.
Simulation results (cont’d)

Figure: Number of users leaving because of lack of coverage as a function of arrival rate.

Figure: Average base station throughput as a function of arrival rate.
The $\alpha$-fair scheduler in the context of OFDMA with MIMO has been analysed.

Scheduling gain for $\alpha \in \{0, 1, +\infty\}$ has been derived using the asymptotic distribution of the MIMO channel capacity.

A mechanism based on $\alpha$-fair schedulers has been introduced and its performance has been evaluated using a network simulator.

The mechanism brings a considerable increase in user perceived quality of service.