On the use of packet scheduling in self-optimization processes: application to coverage-capacity optimization

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Abstract

Self-organizing networks (SON) is commonly seen as a way to increase network performance while simplifying its management. This paper investigates Packet Scheduling (PS) in the context of self-optimizing networks, and demonstrates how to dynamically improve coverage by adjusting the scheduling strategy. We focus on α-fair schedulers, and we provide methods for calculating the scheduling gain, including several closed form formulas. We then propose a coverage-capacity self-optimization scheme based on α-fair schedulers. The method uses Key Performance Indicators (KPIs) to calculate the optimal α in both simple and efficient manner. A simple use case illustrates the implementation of the SON feature and simulation results show the important coverage gains that can be achieved at the expense of very little computing power. [1]

Index Terms

Wireless communication, Self-Optimizing Networks, Scheduling

I. Introduction

Next Generation (NG) Radio Access Networks (RAN) encompassing Beyond 3G (B3G) and 4G networks target ambitious performance and Quality of Service (QoS) objectives. Evolutions in NG RANs are geared by new applications and services with increasing demand for bandwidth and for high QoS while keeping cost and complexity as low as possible. In this context, SON is commonly seen as a key lever to further increase network performance, to simplify its management and to reduce its cost of operation. Main standardization bodies such as 3rd Generation Partnership Project (3GPP) and IEEE have picked up this topic, and SON mechanisms encompassing self-configuration, self-optimization and self-healing are expected to become widely commercially available with the introduction of 4G networks (e.g. LTE Advanced [1] and WiMax 802.16m [2]).

The academic and industrial communities have defined requirements, challenges and many use cases for SON in B3G and 4G RANs (see for example [3], [4], [5]). Self-optimizing network is a SON mechanism that aims at adapting the network to variations in traffic, in propagation conditions and to modification in the operating conditions such as the introduction of a new service. Self-optimizing network has been defined in [1] as "the process where User Equipment (UE) and enhanced eNode B (eNB) measurements and performance measurements are used to auto-tune the network ". Ref. [4] is probably the most complete document providing the requirements and use cases for SON in general and self-optimizing network in particular. Among the self-optimizing use cases are: interference control, handover parameter optimization, parameter optimization of Radio Resource Management (RRM) functionalities governing QoS such as admission control, congestion control, packet scheduling, load balancing, link level retransmission scheme optimization etc. An important SON use case for network operators is the outage detection and compensation ([4], [6]).

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This paper investigates PS in the context of self-optimizing networks. We believe that a packet scheduler can serve as a central tool for designing efficient SON mechanisms in NG RANs. The packet scheduler can be used as part of a stand alone SON entity as described in this work, or in conjunction with other coordinated SON entities such as mobility and/or Inter-Cell Interference Coordination (ICIC) SON entities. The first challenge we face is to model the PS in time scales in which the self-optimization processes operate which could vary from a hundred milliseconds to tens of seconds and more. The packet scheduling operates on a time scale of a millisecond to react to fast fading. Hence one needs to quantify the scheduling gain which depends on the current traffic distribution, i.e. the number of mobiles and their Signal to Interference plus Noise (SINR) at a long time scale. Furthermore, the computation of the scheduling gain should be performed very rapidly to allow the incorporation of the scheduler in a network simulator used to design the self-optimization functionality.

We focus on the family of α-fair schedulers introduced by [7] which includes well-studied schedulers such as Proportional Fair (PF), Max Throughput (MTP) and Max-Min Fair (MMF) schedulers. A general framework for calculating a scheduling gain has been proposed in [8] for $\alpha = 1$ and is generalized to $\alpha > 0$ in this work. For certain particular cases, namely MMF and MTP, we provide closed form expressions for the scheduling gain. These expressions allow us to understand what kind of scheduling gain can be achieved in limit cases for α and its potential use for improving QoS.

We demonstrate the application of the statistical-based scheduler gain calculation to the family of α-fair schedulers that will be used in the self-optimization process. It is noted that deriving the scheduling gain is done without considering the Rayleigh fading time series obtained by the Jakes Model ([9], [10]). The reason is relatively simple: if the scheduling interval is large enough, the channel states at different scheduling times are independent (see II-B), which makes the calculation a lot simpler.

The second contribution of the paper is to derive a coverage-capacity self-optimization scheme based on the family of α-fair schedulers. We show how to adjust the scheduling strategy dynamically to maximize the cell coverage while minimizing the corresponding capacity losses measured in terms of global cell throughput. The self-optimization scheme uses a strategy inspired by the Multi-Armed-Bandit (MAB) problem to learn the optimal α dynamically. The proposed approach is simple and computationally efficient and can serve as basis for real implementation.

The paper is organized as follows: Section II provides the definition of the α-fair scheduler as a maximization problem, and the model chosen for fast-fading. We state an explicit scheduling rule and it's heuristic justification. Section III demonstrates rigorously that this scheduling rule solves the maximization problem, using stochastic approximation techniques. Section IV deals with how to calculate the scheduling gain for a given α, with several closed form formulas for particular values of α and a numerical method for the remaining cases. In section V we examine several simulation results to show the behavior of the scheduling gain when α varies, and how we might take advantage of it to manage fairness dynamically. Section VI describes how the computation of the scheduler gain can be used for capacity-coverage self-optimization in an environment with varying traffic and provides numerical results. Section VII concludes the paper.

II. Model and Assumptions

A. α-fair scheduling

1) Definition: We consider a cell with N users with no mobility, and we adopt a full buffer traffic model. We are considering the downlink scheduling, and the scheduler picks a user for transmission at regular time intervals. A scheduling policy P is defined by the choice of a user for every scheduling instant $(P_{t_m})_{m\in\mathbb{N}}$, namely $P_{t_m} = i$ means that at time t_m, user i will be selected for transmission. We define r_{i,t_m} the instantaneous throughput of user i at time t_m, and $\overline{r}_{i,t_m}$ the mean throughput allocated to user i during the time interval $[t_0, t_m]$. We assume perfect channel knowledge, that is to say that at t_m, the scheduler knows r_{i,t_m} $\forall i$ and can make use of it to choose the scheduled user. Let $\epsilon > 0$ denote a small averaging parameter, define $\overline{r}_{i,t_m}$ as in [11] by the following recursive equation:

$$\overline{r}_{i,t_{m+1}} = (1-\epsilon)\overline{r}_{i,t_m} + \epsilon\delta_{P_{t_{m+1}},i} r_{i,t_{m+1}} \tag{1}$$

where δ denotes Kronecker's delta. ϵ controls the size of the averaging window, and for voice traffic, the play-out buffer size is 80ms, which implies that we should choose ϵ so that $(1-\epsilon)^{80}$ is small, for example $\epsilon = 0.05$. This

definition for the mean allocated throughput is more relevant to reflect the QoS perceived by a user than using an arithmetic mean (which would be replacing ϵ in (1) by $\frac{1}{m}$) because it induces a "decay" of past observed values. If we assume that $\overline{r}_{i,t_0} = 0 \ \forall i$, equation (1) can also be written:

$$\overline{r}_{i,t_m} = \epsilon \sum_{j=0}^{m} (1-\epsilon)^{m-j} \delta_{P_{t_j},i} r_{i,t_j} \tag{2}$$

Furthermore, if $\overline{r}_{i,t_m}$ has a limit when $m \to +\infty$, $\epsilon \to 0^+$, we then denote by $\overline{r}_{i,+\infty}$ this limit.

We also make the assumption that $(r_{i,t_m})_{m\in\mathbb{N}}$ is an i.i.d sequence for all i, and that $(r_{i,t_m})_{m\in\mathbb{N}}$ is independent of $(r_{k,t_m})_{m\in\mathbb{N},k\neq i}$.

As introduced in [7], the α-fair scheduler for $\alpha \in [0,+\infty)$ is the policy that maximizes the following utility function, given the time interval $[t_0, t_M]$:

$$U = \begin{cases} \displaystyle\sum_{i=1}^{N} \log(d + \overline{r}_{i,t_M}) & , \ \alpha = 1 \\ \displaystyle\sum_{i=1}^{N} \frac{(\overline{r}_{i,t_M} + d)^{1-\alpha}}{1-\alpha} & , \ \alpha \neq 1 \end{cases} \tag{3}$$

where $d > 0$ can be chosen as small as desired and is only present to avoid problematic behavior near 0.

2) Allocation: We now give a heuristic justification for the scheduling rule, and a rigorous analysis is given in Section III. We assume that the allocation has been done for $[t_0, t_M]$ and we want to decide which user to schedule at t_{M+1}. Let $(\Delta U)_i$ denote the variation of utility if user i is chosen for transmitting at t_{M+1}, which we approximate with a first-order Taylor expansion. If $\alpha = 1$, the increase in utility for user i is:

$$\begin{aligned} &\log\left((1-\epsilon)\overline{r}_{i,t_M} + \epsilon r_{i,t_{M+1}} + d\right) - \log(\overline{r}_{i,t_M} + d) \\ &\qquad = \epsilon \frac{r_{i,t_{M+1}} - \overline{r}_{i,t_M}}{\overline{r}_{i,t_M} + d} + o(\epsilon) \end{aligned} \tag{4}$$

The decrease for the other users is:

$$\log\left((1-\epsilon)\overline{r}_{i,t_M}\right) - \log(\overline{r}_{i,t_M}) = -\epsilon \frac{\overline{r}_{i,t_M}}{\overline{r}_{i,t_M} + d} + o(\epsilon) \tag{5}$$

We add (4) and (5):

$$(\Delta U)_i = \epsilon \left[\frac{r_{i,t_{M+1}}}{\overline{r}_{i,t_M} + d} - \sum_{k=1}^{N} \frac{\overline{r}_{k,t_M}}{\overline{r}_{k,t_M} + d} \right] + o(\epsilon) \tag{6}$$

If $\alpha \neq 1$:

$$\begin{aligned} &\frac{1}{1-\alpha} \left[\left((1-\epsilon)\overline{r}_{i,t_M} + \epsilon r_{i,t_{M+1}} + d\right)^{1-\alpha} - (\overline{r}_{i,t_M} + d)^{1-\alpha} \right] \\ &\qquad = \epsilon \frac{r_{i,t_{M+1}} - \overline{r}_{i,t_M}}{(\overline{r}_{i,t_M} + d)^{\alpha}} + o(\epsilon) \end{aligned} \tag{7}$$

and:

$$\begin{aligned} &\frac{1}{1-\alpha} \left[((1-\epsilon)\overline{r}_{i,t_M} + d)^{1-\alpha} - (\overline{r}_{i,t_M} + d)^{1-\alpha} \right] \\ &\qquad = -\epsilon \frac{\overline{r}_{i,t_M}}{(\overline{r}_{i,t_M} + d)^{\alpha}} + o(\epsilon) \end{aligned} \tag{8}$$

We add (7) and (8):

$$(\Delta U)_i = \epsilon \left[\frac{r_{i,t_{M+1}}}{(\overline{r}_{i,t_M} + d)^{\alpha}} - \sum_{k=1}^{N} \frac{\overline{r}_{k,t_M}}{(\overline{r}_{k,t_M} + d)^{\alpha}} \right] + o(\epsilon) \tag{9}$$

In both cases, for small ϵ, the optimal choice is:

$$i^* = \arg\max_{0\leq i\leq N} \frac{r_{i,t_{M+1}}}{(\overline{r}_{i,t_M} + d)^{\alpha}} \tag{10}$$

$\alpha = 1$ corresponds to a PF scheduler, and $\alpha = 0$ to a MTP scheduler.

B. Channel Model

Let c_{i,t_m} be the instantaneous channel quality for user i at time t_m, that is to say the product of path loss, shadowing and fast fading. We will assume that $r_{i,t_m} = \Phi(c_{i,t_m})$. Φ is a function that maps channel quality into bit-rate, and is given in the form of a quality table obtained from a link level simulator. Φ captures the effect of physical layer mechanisms such as modulation, coding and retransmissions. This function therefore depends on the technology we are considering, which is Time Division Multiple Access (TDMA) technology such as High Speed Downlink Packet Access (HSDPA).

Let us denote by S_i the average SINR for user i, which captures the effect of path loss, shadowing and interference with neighboring cells. We choose a time scale that is short enough for all those effects to be constant, but long enough to capture a scheduling gain, and the only random parameter is the fast fading ξ.

The channel fading is described by a Rayleigh model, and we use the assumption from [12] that the number of interfering signals is sufficiently large so that the fading processes between users and neighboring cells base stations can be ignored for the calculation of S_i. The instantaneous channel quality can then be written as:

$$c_{i,t_m} = S_i \xi_{i,t_m} \quad , \quad \xi_{i,t_m} \equiv Exponential(1) \tag{11}$$

Furthermore, random variables $(\xi_{i,t_m})_{0 \leq i \leq N, m \in \mathbb{N}}$ are independent. Independence between users comes from the Rayleigh fading model, and independence between different instants is verified if $t_{m+1} - t_m$ is larger than the channel coherence time. More precisely, as stated in [9], the autocorrelation of the channel fading for a single user between t and $t + \tau$ is $J_0(\omega_M \tau)$, where J_0 is the 0-th order Bessel function and ω_M-the maximum Doppler shift, and $|J_0(x)|$ vanishes as x grows.

III. Convergence Analysis

In this section we give a convergence analysis of α-fair scheduling, using the Ordinary Differential Equation (ODE) technique which has been used previously in [13] and [11] to show the convergence of the PF scheduler.

A. Stochastic approximation

We start by giving two results from stochastic approximation theory, which links the behavior of stochastic iterative algorithms with limit sets of a certain ODE. We consider $\theta \in \mathbb{R}^n$, $(a, b) \in \mathbb{R}^n \times \mathbb{R}^n$, $H = \{x \in \mathbb{R}^n | a_i \leq x_i \leq b_i\}$, $\Pi_H[x] = \underset{y \in H}{\operatorname{argmin}} \, ||x - y||$, step sizes $\epsilon_k > 0$ and random variables $Y_k(\theta) \in \mathbb{R}^n$. We assume that the Y_k are independent and identically distributed (i.i.d) with $\mathbb{E}[Y_k(\theta)] = g(\theta)$ and $\sup_\theta \mathbb{E}[Y_k(\theta)^2] < +\infty$. We define the sequence θ_k using the following algorithm:

$$\theta_{k+1} = \Pi_H[\theta_k + \epsilon_k Y_k] \tag{12}$$

Two choices for the step sizes are possible:

(Pi) $\epsilon_k > 0$, $\sum_{k \geq 1}^{+\infty} \epsilon_k = +\infty$, $\sum_{k \geq 1}^{+\infty} \epsilon_k^2 < +\infty$ which is adequate when the environment is stationary, and ensures a strong form of convergence as shown below

(Pii) $\epsilon_k = \epsilon > 0$ where ϵ is a small constant, which allows following slow variations of the environment

We will also assume that g is continuous, that the mean ODE $\dot{\theta} = g(\theta)$, $\theta(0) = \theta_0$ has a unique solution defined on $\mathbb{R}^+$ for all θ_0, and that all solutions converge to θ^* in the interior of H. Before stating the theorems it shall be noted that the assumptions we have made are extremely restrictive in order to make the theorems statements less technical, and that a lot of other cases can be handled by stochastic approximation theory, including non i.i.d variables, cases in which the mean ODE does not converge to a single point and when it is replaced by a differential inclusion.

The asymptotic behavior of (12) is given by the following theorems:

Theorem 1. *If we assume (Pi) then* $\theta_k \underset{k \to +\infty}{\to} \theta^*$ *almost surely.*

Theorem 2. *If we assume (Pii) then there exists a constant* $K_1 > 0$ *such that* $\limsup_{k \to +\infty} \mathbb{E}[||\theta_k - \theta^*||^2]^{\frac{1}{2}} \leq K_1 \sqrt{\epsilon}$

The first theorem is implied by [13] (Theorem 2.1, page 127) and the second is [14] (Theorem 3, Page 106). Intuitively, the second theorem states that we can always find an ϵ so that the accumulation points of the sequence

θ_k are almost all the time in an arbitrarily small neighborhood of the limit point θ^*, giving a form of convergence in distribution (or weak convergence).

B. Application to the α-fair scheduler, $\alpha > 0$

We will now use the previous results to show that the α-fair scheduler defined by (10) converges to a unique limit, and that it maximizes the utility function (3). We work with $\alpha > 0$ fixed, and the case $\alpha = 0$ will be studied separately.

We use the following notation: $(x, y) \in \mathbb{R}^n \times \mathbb{R}^n$, $x \leq y \Leftrightarrow x_i \leq y_i$, $1 \leq i \leq n$. The scheduling rule (10) has the form (12), with θ_k the mean throughput at time k, ϵ a small constant, and $g(\theta) = h(\theta) - \theta$, where h is defined by: $h(\theta) = \mathbb{E}[rI_{\arg\max(\frac{r}{(d+\theta)^\alpha})}]$ with $(I_i)_k = \delta(i,k)$, $1 \leq k \leq n$. We will assume that r is always positive with $\mathbb{E}(r) = \overline{r} < +\infty$ and $\mathbb{E}(r^2) < +\infty$ .We also assume that r has a density with respect to the Lebesgue measure on $(\mathbb{R}^+)^n$, and that its components are independent. It shall be noted that those assumptions are not very restrictive and are satisfied for Rayleigh and Rice fading models.

1) Properties of h: We have that h is positive and bounded, since $h \leq \overline{r}$.

We have that if $h(\theta_1) = \theta_1$, $\theta_1 \leq \theta_2$ and $h(\theta_2) = \theta_2$ then $\theta_1 = \theta_2$, since all components of h cannot increase when all components of θ increase.

We are going to prove that h is also Lipschitz continuous. We first assume that $||\theta|| < 1$, let $P_{i,j,\theta_1,\theta_2}$ be the following quantity:

$$P_{i,j,\theta_1,\theta_2} = \mathbb{P}\left[\left\{\frac{r_i}{(d+\theta_{1_i})^\alpha} \geq \frac{r_j}{(d+\theta_{1_j})^\alpha}\right\} \cup \left\{\frac{r_i}{(d+\theta_{2_i})^\alpha} \leq \frac{r_j}{(d+\theta_{2_j})^\alpha}\right\}\right]$$

which we can rewrite:

$$P_{i,j,\theta_1,\theta_2} = \mathbb{P}\left[r_j(\frac{d+\theta_{1_i}}{d+\theta_{1_j}})^\alpha \leq r_i \leq r_j(\frac{d+\theta_{2_i}}{d+\theta_{2_j}})^\alpha\right] \tag{13}$$

Let $F_{r_i}(x) = \mathbb{P}[r_i \leq x]$,

$$P_{i,j,\theta_1,\theta_2} = \mathbb{E}\left[F_{r_i}(r_j(\frac{d+\theta_{1_i}}{d+\theta_{1_j}})^\alpha) - F_{r_i}(r_j(\frac{d+\theta_{2_i}}{d+\theta_{2_j}})^\alpha)\right] \tag{14}$$

We have assumed $||\theta|| \leq 1$, so we have :

$$|(\frac{d+\theta_{1_i}}{d+\theta_{1_j}})^\alpha - (\frac{d+\theta_{2_i}}{d+\theta_{2_j}})^\alpha| \leq K_\alpha||\theta_1 - \theta_2|| \tag{15}$$

F_{r_i} is Lipschitz since we have assumed r_i to have a density with respect to the Lebesgue measure, so for a certain constant K_F:

$$P_{i,j,\theta_1,\theta_2} \leq \mathbb{E}[K_\alpha K_F||\theta_1 - \theta_2||r_j] = K_\alpha K_F||\theta_1 - \theta_2||\overline{r}_j \tag{16}$$

We now apply Cauchy-Schwartz inequality to evaluate the variation of h

$$\begin{aligned}||h(\theta_1) - h(\theta_2)|| \leq & \mathbb{E}[||r||^2]\mathbb{E}[||I_{\arg\max(\frac{r}{(d+\theta_1)^\alpha})} \\ & - I_{\arg\max(\frac{r}{(d+\theta_2)^\alpha})}||^2]\end{aligned} \tag{17}$$

The first term is finite since we have assumed finite variance for r and the second term can be evaluated by:

$$\mathbb{E}[||I_{\arg\max(\frac{r}{(d+\theta_1)^\alpha})} - I_{\arg\max(\frac{r}{(d+\theta_2)^\alpha})}||^2] \leq 4\sum_{i\neq j} P_{i,j,\theta_1,\theta_2} \tag{18}$$

Combining (17) and (18) we conclude that there exists C_h constant so that:

$$||h(\theta_1) - h(\theta_2)|| \leq C_h||\theta_1 - \theta_2|| \tag{19}$$

We therefore have proved that h is Lipschitz for $||\theta|| \leq 1$. Let $K_2 \geq 1$, we have that :

$$I_{\arg\max(\frac{r}{(d+\theta)^\alpha})} = I_{\arg\max(\frac{r}{(\frac{d+\theta}{K_2})^\alpha})} \tag{20}$$

and therefore:

$$h(\theta) = h(\frac{\theta + d}{K_2} - d) \tag{21}$$

We combine this with (19), with K_2 large enough:

$$\begin{aligned} ||h(\theta_1) - h(\theta_2)|| &= ||h(\frac{\theta_1 + d}{K_2} - d) - h(\frac{\theta_2 + d}{K_2} - d)|| \\ &\leq \frac{C_h}{K_2}||\theta_1 - \theta_2|| \\ &\leq C_h||\theta_1 - \theta_2|| \end{aligned}$$

So we have proved that h is globally Lipschitz continuous.

2) Existence of a solution to the ODE: We now have to prove that the ODE has solutions on $\mathbb{R}^+$. We have that h is Lipschitz, so the Picard-Lindelof theorem assures us that it has a unique local solution. Furthermore, we know that there exists a unique maximal solution defined on some maximal interval $[0, t_0[$. h is bounded by $\overline{r}$ so $\theta(t) \leq \theta(0) + t\overline{r}$, therefore $t_0 = +\infty$, or else the solution is not maximal.

3) Monotone dynamical systems: We first state some results from the theory of monotone dynamical systems, and the reader can refer to [15] for their proofs.

We denote by $\Phi_t(x)$, $x \in (\mathbb{R}^+)^n$ the value at time t of the solution to the ODE starting in x. We define the orbit of x by $O(x) = \{\Phi_t(x) | t \geq 0\}$ and the limit set of x by $\omega(x) = \cap_{t\geq 0} \cup_{s\geq t} \Phi_s(x)$. x is called an equilibrium point if $O(x) = x$, and we denote by E the set of equilibrium points. x is called a quasi-convergent point if $\omega(x) \subset E$ and we denote by Q the set of quasi-convergent points. If $x \leq y \Rightarrow \Phi_t(x) \leq \Phi_t(y)$ $\forall (x, y) \in (\mathbb{R}^+)^n \times (\mathbb{R}^+)^n$ $\forall t \in \mathbb{R}^+$, then we say that Φ is monotone. We have the following theorems:

Theorem 3. *If Φ is monotone and $x < y$ then either:*

(i) $\omega(x) < \omega(y)$ *, or*
(ii) $\omega(x) = \omega(y) \subset E$

Theorem 4. *If Φ is monotone then Q is dense in $(\mathbb{R}^+)^n$*

We now need to show that those results can be applied to the ODE we are considering, which is proved by the following comparison theorem:

Theorem 5. *We consider the ODE $\dot{x}= g(x)$. Let $g : (\mathbb{R}^+)^n \to \mathbb{R}^n$, verifying:*

(i) g is continuous
(ii) The solution to the ODE is unique for every initial condition
(iii) $x \leq y$ and $x_i = y_i \Rightarrow g_i(x) \leq g_i(y)$
(iv) For $T \geq 0$, $(x, \delta) \in (\mathbb{R}^+)^n \times (\mathbb{R}^+)^n$, we have that: $\sup_{0\leq t\leq T} ||\Phi_t(x) - \Phi_t(x + \delta)|| \underset{\delta\to 0}{\to} 0$

Then Φ is monotone

Condition (iii) is often called the Kamke condition.

Let us now show that the ODE we are considering satisfies those conditions. (i) and (ii) have been proved previously. (iii) comes from the fact that $x \to \frac{1}{(d+x)^\alpha}$ is decreasing. To prove (iv), let $T > 0$, since h is Lipschitz we can apply Gronwall's lemma:

$$||\Phi_t(x) - \Phi_t(x + \delta)|| \leq ||\delta||e^{K_3 t} \tag{22}$$

for a certain constant K_3. We then have that:

$$\sup_{0\leq t\leq T} ||\Phi_t(x) - \Phi_t(x + \delta)|| \leq ||\delta||e^{K_3 T} \underset{\delta\to 0}{\to} 0 \tag{23}$$

So the conditions of the previous theorem are valid.

4) Convergence for $\theta(0) = 0$: By noticing that $g(0) > 0$, the following theorem proves that the solution starting at 0 converges to a certain θ^*.

Theorem 6. *If the ODE verifies the Kamke condition then any solution starting at x with $g(x) > 0$ converges to an equilibrium point.*

Let us now show that all solutions converge to the same limit.

We have proved that $\omega(0) = \{\theta^*\}$. Let $x > 0$ be an arbitrary initial condition, and $x_1 \geq x$ with $x_1 \in Q$ since Q is dense in $(\mathbb{R}^+)^n$. We know that $\omega(x_1) \subset E$ since $x_1 \in Q$, let us assume that $\omega(0) < \omega(x_1)$. Let $x_2 \in \omega(x_1)$, we have that $h(x_2) = 0$ and $x_2 > \theta^*$, which contradicts (III-B1). So $\omega(x_1) = \omega(0) = \{\theta^*\}$, and finally $\omega(x) = \{\theta^*\}$ $\forall x \leq 0$, in other words all solutions converge to θ^*.

5) Optimality: Finally, we have to prove that the scheduling strategy is optimal, namely that any other scheduling strategy achieves lower utility.

We first differentiate the utility function:

$$\dot{U}\ (\theta(t)) = \sum_{i=1}^{n} \frac{h_i(\theta(t)) - \theta_i(t)}{(d + \theta_i(t))^\alpha} \tag{24}$$

We are going to prove that θ^* is a local maximum of U on the set of all achievable throughputs. Let f : $(\mathbb{R}^+)^n \times (\mathbb{R}^+)^n \to \{1, ..., n\}$ a new allocation rule, by replacing h by it's definition we have that:

$$\mathbb{E}[\sum_{i=1}^{n} \frac{r_i(I_{f(\theta,r)})_i}{(d + \theta_i(t))^\alpha}] \leq \mathbb{E}[\sum_{i=1}^{n} \frac{r_i(I_{\arg\max(\frac{r}{(d+\theta)^\alpha})})_i}{(d + \theta_i(t))^\alpha}]. \tag{25}$$

Let $\theta_f(t)$ and $\theta(t)$ the trajectories implied by the new and the usual allocation rules respectively, both starting at θ^*. By combining (24) and (25) at $t = 0$ we have that:

$$\dot{U}\ (\theta_f(t))|_{t=0} \leq \dot{U}\ (\theta(t))|_{t=0} \leq 0 \tag{26}$$

Therefore θ^* is a local maximum of U on the set of all achievable throughputs.

Now consider θ_m achievable and such that $U(\theta_m) > U(\theta^*)$. There is a certain allocation policy f such that $\theta_m = \mathbb{E}[rI_{f(r)}]$. Starting at θ^* and using the allocation f gives the ODE $\dot{\theta}= \theta_m - \theta$, the solution being $\theta(t) = e^{-t}\theta^* + (1 - e^{-t})\theta_m$. Since $\alpha > 0$, U is strictly concave, and it must be strictly increasing at the beginning of this path, which contradicts the fact that θ^* is a local maximum. We therefore have proved that the scheduling rule achieves optimal utility.

C. Application to the α-fair scheduler, $\alpha = 0$

The case $\alpha = 0$ is a bit different since U is linear, and not strictly concave. However the proof is a lot simpler since the scheduling strategy does not depend on the mean throughput. The ODE is $\dot{\theta}= \mathbb{E}[rI_{\arg\max(r)}] - \theta$, and the solution is $\theta(t) = e^{-t}\theta_0 + (1 - e^{-t})\mathbb{E}[rI_{\arg\max(r)}]$, which converges to an unique limit. It shall be noted that the limit is unique because $\mathbb{P}[r_i = r_j, i \neq j] = 0$. If $\mathbb{P}[r_i = r_j, i \neq j] > 0$ it might not be the case, for example consider the case where all the r_i are constant and equal to 1, any point in the simplex is a limit throughput. It is also easy to see that since we have assumed independence of the channel between two scheduling instants and that U is linear, the policy that chooses the user with the best channel also maximizes U over the set of achievable throughputs.

IV. Scheduling gain

A. General expression

Let $\overline{r}_{i,+\infty,\alpha}$ denote the mean limit throughput allocated to user i by an α-fair scheduler, and $\overline{r}_{i,+\infty,RR}$ the same quantity for a Round Robin (RR) scheduler. It is noted that $\overline{r}_{i,+\infty,\alpha}$ is well-defined according to the convergence analysis done in Section III. We use the RR scheduler as a reference, and we want to calculate the scheduling gain of an α-fair scheduler $G_\alpha = \frac{\overline{r}_{i,+\infty,\alpha}}{\overline{r}_{i,+\infty,RR}}$.

For a given α, the scheduling strategy (10) converges to a unique limit θ^* with $h(\theta^*) = \theta^*$, and combining this with the channel model, yields the following integral equation:

$$\overline{r}_{i,+\infty,\alpha} = \int_0^{+\infty} \Phi(x) \mathbb{P}\left[\frac{\Phi(x)}{\overline{r}^{\alpha}_{i,+\infty,\alpha}} \geq \max_{k\neq i}\left(\frac{\Phi(S_k\xi_k)}{\overline{r}^{\alpha}_{k,+\infty,\alpha}}\right)\right] \frac{e^{\frac{-x}{S_i}}}{S_i} dx \tag{27}$$

It is important to notice that this formula in its present form does not enable us to calculate the scheduling gain, since we need to know the value of $\overline{r}_{k,+\infty,\alpha}$ $\forall k$. We will now show some particular cases where analytic formulas exist, and give a numerical method for other cases.

B. RR

Let L_Φ denote the Laplace transform of Φ, the RR scheduler chooses a given user with probability $\frac{1}{N}$, which gives:

$$\begin{aligned}\overline{r}_{i,+\infty,RR} &= \frac{1}{N}\int_0^{+\infty} \frac{\Phi(x)}{S_i} e^{\frac{-x}{S_i}} dx \\ &= \frac{1}{NS_i} L_\Phi\left(\frac{1}{S_i}\right)\end{aligned} \tag{28}$$

C. PF

Results for the PF case ($\alpha = 1$) are given in [12]:

$$\overline{r}_{i,+\infty,1} = \frac{1}{S_i}\sum_{k=0}^{N-1}\binom{N-1}{k}(-1)^k L_\Phi\left(\frac{k+1}{S_i}\right) \tag{29}$$

In particular, if Φ is a linear function we have the simple expression [8]:

$$G_{i,1} = \sum_{k=1}^{N}\frac{1}{k} \tag{30}$$

which is asymptotically equivalent to $\log(N)$.

D. MTP

Let us examine the case of a MTP, that is $\alpha = 0$. The probability of choosing user i is the probability that he has the best channel quality:

$$\mathbb{P}\left[x > max_{k\neq i}(c_{k,t})\right] = \prod_{k\neq i}(1 - e^{-\frac{x}{S_i}}) \tag{31}$$

The throughput is then

$$\overline{r}_{i,+\infty,0} = \int_0^{+\infty} \Phi(x) \prod_{k\neq i}(1 - e^{-\frac{x}{S_k}}) \frac{e^{-\frac{x}{S_i}}}{S_i} dx \tag{32}$$

By developing the product, we obtain the following expression:

$$\overline{r}_{i,+\infty,0} = \frac{1}{S_i}\sum_{k=0}^{N-1}(-1)^k \sum_{a_1<\ldots<a_k, a_j\neq i\ \forall j} L_\Phi\left(\frac{1}{S_i} + \sum_{j=0}^{k}\frac{1}{S_{a_j}}\right) \tag{33}$$

E. MMF

We now consider the case $\alpha \to +\infty$, which is analytically tractable, and of particular interest for the dynamic alpha policy we will address later.

Since for $m > 1$, $\mathbb{P}\left[\overline{r}_{i,t_m} = \overline{r}_{j,t_m}\ ,\ i \neq j\right] = 0$ and $\mathbb{P}\left[r_{i,t_m} = 0\right] = 0$, the scheduling rule (10) becomes:

$$i^* = \arg \min_{0 \leq i \leq N} (\overline{r}_{i,t_m}) \tag{34}$$

We are now going to prove that all users have the same throughput for a MMF scheduler. We know that $\overline{r}_{i,+\infty,+\infty}$ exists, and let us assume that $\overline{r}_{i,+\infty,+\infty} > \overline{r}_{j,+\infty,+\infty} \geq 0$, therefore, there exists a T so that:

$$\overline{r}_{i,t_m,+\infty} > \overline{r}_{j,t_m,+\infty} \quad t_m \geq T \tag{35}$$

which means that user i never transmits after T, and so $\overline{r}_{i,+\infty,+\infty} = 0$ which contradicts our initial assumption. Therefore, $\overline{r}_{i,+\infty,+\infty} = \overline{r}_{j,+\infty,+\infty}\ \forall i, j$.

We know that when user i is alone in the cell, it's throughput is:

$$\int_0^{+\infty} \Phi(x) \frac{e^{-\frac{x}{S_i}}}{S_i} dx = \frac{1}{S_i} L_\Phi(\frac{1}{S_i}) \tag{36}$$

and since the scheduling rule (34) does not depend on the instantaneous throughput, we have that $\overline{r}_{i,+\infty,+\infty} = p_i \frac{1}{S_i} L_\Phi(\frac{1}{S_i})$, with $\sum_{k=1}^N p_k = 1$ and $\overline{r}_{i,+\infty,+\infty} = \overline{r}_{j,+\infty,+\infty}$, $\forall i, j$, which gives the following formula:

$$\overline{r}_{i,+\infty,+\infty} = \frac{1}{\sum_{k=1}^N \frac{S_k}{L_\Phi(\frac{1}{S_k})}} \tag{37}$$

This formula will be useful later, because it enables us to determine analytically which users can be covered by adjusting the α, and which users will never be able to be covered. Scheduling them would simply waste resources and they therefore should be ignored when deciding which α to use.

F. α-fair

For the cases in which the throughput cannot be calculated analytically, we still can use the results from Section III to calculate it numerically. We get the algorithm described in Table I, where T is the number of simulation steps, the $\xi_i(t)$ are independent exponential random variables with mean 1 and ϵ_n are the step sizes.

1. $\overline{r}_{i,t_0,\alpha} = 0\ \forall i$

For t_m **from** t_0 **to** T**:**

2. Draw N exponentially distributed variables $(\xi_i(t_m))_{0 \leq i \leq N}$

3. $i^* = \arg\max_{0 \leq i \leq N} (\frac{S_i \xi_i(t_m)}{\overline{r}^\alpha_{i,t_m,\alpha}})$

4. $\overline{r}_{i,t_{m+1},\alpha} = \begin{cases} (1-\epsilon_n)\overline{r}_{i,t_m,\alpha} + \epsilon_n \Phi(S_i \xi_i(t_m)) & ,\ i = i^* \\ (1-\epsilon_n)\overline{r}_{i,t_m,\alpha} & ,\ i \neq i^* \end{cases}$

TABLE I
NUMERICAL METHOD FOR CALCULATING $\overline{r}_{i,+\infty,\alpha}$

We can choose either $\epsilon_n = \frac{1}{n}$ or $\epsilon_n = \epsilon$ with ϵ a small constant. As stated in Section III, convergence to $\overline{r}_{i,+\infty,\alpha}$ occurs in both cases. There is almost sure convergence in the first case and weak convergence in the latter.

G. Remark

It shall be noted that, while the method given for the MTP scheduler is analytically tractable, if $S_i \neq S_j\ \forall i \neq j$ the number of terms to evaluate in (33) is 2^{N-1}, and therefore the formula can only be used for small values of N, in the order of $N \leq 15$. For larger values we will have to rely on the numerical method instead.

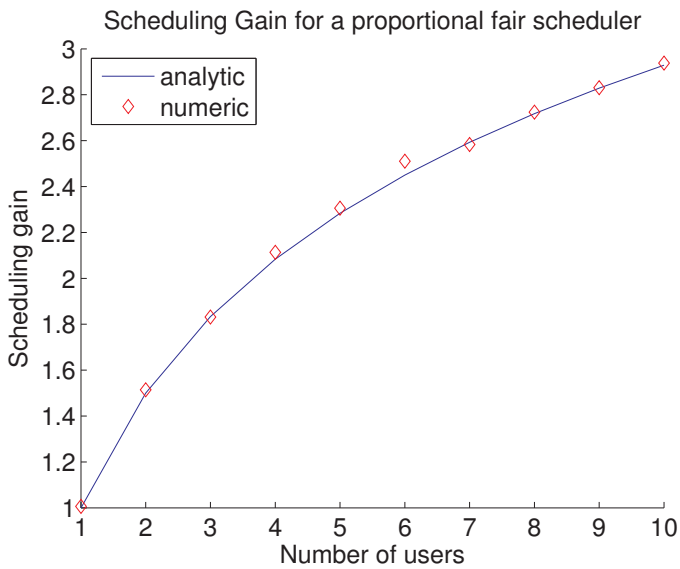

Fig. 1. PF scheduling gain as a function of the number of users for $S_i = 1\ \forall i$

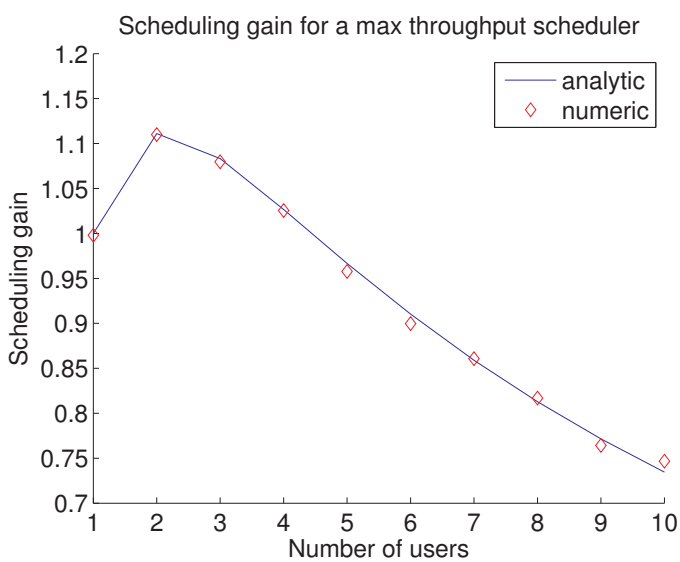

Fig. 2. MTP scheduling gain as a function of the number of users for user 1 with $S_1 =$ and $S_i = 2$ for $i \geq 2$

V. Behavior of different scheduling strategies

A. Scenarios

Three scenarios have been simulated, and for all of them we take $\Phi(x) = x$ for simplicity, which corresponds to the infinite bandwidth case.

Scenario 1: PF scheduler, N users with $S_i = 1\ \forall i$, since in this particular case the gain is insensitive to the S_i, it is the same for all users.

Scenario 2: MTP scheduler, N users with $S_1 = 1$, $S_i = 2$ for $i \geq 2$. We are interested in the gain of the first user. The gain is not the same for everyone since the scheduler is relatively "unfair".

Scenario 3: α-fair scheduler, 2 users with $S_1 = 1$, $S_2 = 10$. The point of this scenario is to illustrate what happens when a user is near the base station and the other one is far. By far we mean that either the user is physically far from the BS, or he is in an area with very deep shadow fading. In both cases this user will have a bad average channel quality.

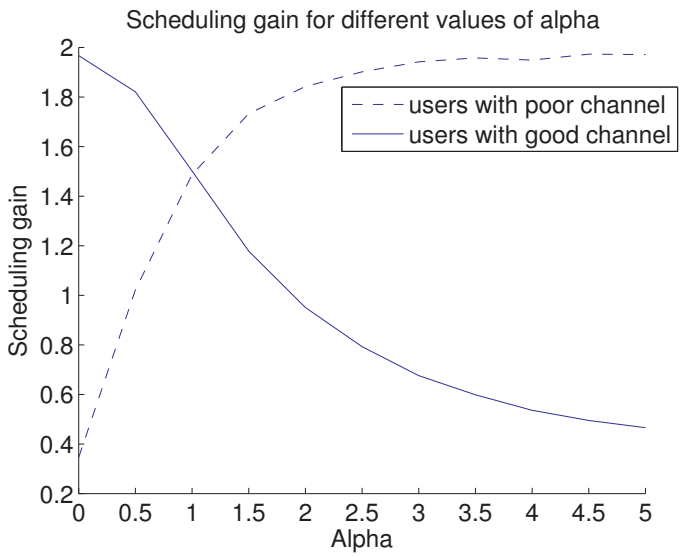

Fig. 3. Scheduling gain as a function of α for 2 users and $S_1 = 1$, $S_2 = 10$

B. Interpretation

Figure 1 shows the scheduling gain for user 1 in scenario 1, and Figure 2 in scenario 2. We can see on both figures that the numerical method approximates the closed form formulas quite well, and we can also see on Figure 2 that the gain for user 1 decreases when N increases, since he has poorer channel conditions. Figure 3 shows the gain for both users in scenario 3, and we can see that the larger α is, the larger is the gain for users with poor channel conditions, and so it is possible to manage the coverage for users at cell edge by adjusting α dynamically.

VI. Coverage-capacity self-optimization use case

A. System Model

This Section considers an important SON use case, namely the coverage-capacity self-optimization using the above results. We consider a TDMA system such as HSDPA. Mobility is ignored.

1) Time Scales: We call averaging period the interval $[t_m, t_{m'}]$, on which the average throughput given by the scheduler is calculated and determines which users are not covered. Therefore the averaging period should be long enough for the scheduling algorithm to converge, since for our scheduling gain calculations we have assumed a large number of scheduling intervals.

2) Path Loss: Path loss is given by the following formula:

$$L_{i,s} = A\frac{1}{d_{i,s}^{\nu}} \tag{38}$$

with $d_{i,s}$ being the distance between user i and the base station s, and A, ν two constants that depend on the environment.

3) Shadowing: Let $\chi_{i,s}$ denote the shadowing between user i and base station s, which we model by a log-normal random variable:

$$\chi_{i,s} = 10^{\frac{a\epsilon_1 + b\epsilon_2}{10}} \tag{39}$$

with $\epsilon_i \equiv N(0, \sigma^2)$, $i \in \{1, 2\}$ and a,b two constants. As mobility is not considered, shadowing and path loss remain constant during the whole process.

4) Interference: We consider first-tier neighboring cells as the only source of interference and assume that the total interference, I_{tot}, is related to the average neighboring cell load. Let ρ denote the neighboring cell load, we consider here a simple model to show that the scheduler is able to adapt itself to varying traffic conditions (t in ms):

$$\rho(t) = |sin(\pi \frac{t}{300})| \tag{40}$$

(40) assumes that the number of interferers is large.

Interference to user i caused by neighbors follows the same model than the useful signal:

$$I_{i,neighbor} = \rho P_{max} A \frac{\chi_{i,neighbor}}{d^{\nu}_{i,neighbor}} \tag{41}$$

where P_{max} is the maximal power emitted by a base station, and we assume that all base stations emit with the same power.

$$I_{i,tot} = \sum_{neighbor} I_{i,neighbor} \tag{42}$$

5) SINR: Let s_i be the serving base station for user i. The SINR for user i can then be calculated by the following formula:

$$SINR_i = \frac{P_{max} \chi_{i,s_i} L_{i,s_i}}{I_{i,tot} + {\sigma_N}^2} \tag{43}$$

where ${\sigma_N}^2$ is the thermal noise.

6) Coverage: We choose the following definition for coverage: a user is considered covered if his mean throughput during the averaging period is superior to Th_{min}, with Th_{min} a fixed threshold necessary to provide a minimal QoS. We are hence concerned with choosing α properly so that the number of covered users is maximal without degrading the cell capacity.

B. Control strategy

We now get to the main point of the article: designing a self-optimizing network functionality for coverage-capacity optimization. It shall adjust α dynamically, based on the observed KPIs available every averaging period: outage, user throughput, etc.

As mentioned before, when $\alpha \to \infty$, the scheduler becomes a MMF scheduler. Therefore the quantity $\overline{r}_{i,\infty,\infty}$ defines two possible behaviors:

- $\overline{r}_{i,\infty,\infty} > Th_{min}$: if we set α large enough we shall be able to cover user i
- $\overline{r}_{i,\infty,\infty} < Th_{min}$: user i will never be covered, no matter how large α is chosen

Hence, if we are in the latter case, we can use (37) to calculate the throughput of the MMF scheduler, and choose to ignore the users that cannot be covered. This is done by ignoring the user with the biggest $\frac{S_i}{L_\Phi(\frac{1}{S_i})}$, i.e the user with the lowest S_i, since $\frac{L_\Phi(\frac{1}{S_i})}{S_i}$ is the throughput of user i when he is alone in the cell. We can then recalculate the MMF throughput with the formula, and keep doing so until it is above Th_{min}.

C. Optimality criteria

We are interested in finding the α with the best capacity-coverage performance, but we shall not forget that this comes at a price: the larger the α, the larger the capacity loss. For example, choosing $\alpha = +\infty$ all the time would result in covering all users that can be covered all the time, but this controller could hardly be called optimal. Therefore, to avoid a multi-criteria optimization problem, we will define the optimal α as the minimal α that covers all users, and in this way we will not have to consider the maximization of the global throughput explicitly.

D. Modified ϵ-greedy policy

The method proposed here could be seen as a modified version of the ϵ-greedy policy that is popular for several reinforcement learning problems. Thanks to (37), we are able to calculate the maximum number of users that can be covered for α large enough. Therefore any α that has previously resulted in covering all the users that can be covered is an upper bound for the optimal α, at least for a certain period of time if the traffic conditions do not change too drastically.

For the k-th averaging period, n_k denotes the number of covered users and N_k the maximal number of users that can be covered for α large enough. P_ϵ is a small probability used for exploration. The algorithm is described in table II.

Initial phase:
1. Calculate N_0 using (Table III)
2. Try every $\alpha \in \{1, ..., 10\}$ once
3. Choose the minimal α that covers N_0 users.

Repeat:
4. Calculate N_k using (Table III)
5. Set $\alpha = \alpha_k$ and observe resulting n_k

If $n_k < N_k$:
6. $\alpha_{k+1} = \min(\alpha_k + 1; 10)$

If $n_k = N_k$:
7. $\alpha_{k+1} = \begin{cases} \max(\alpha_k - 1; 1) \text{ with probability } P_\epsilon \\ \alpha_k \text{ with probability } (1 - P_\epsilon) \end{cases}$

TABLE II
MODIFIED ϵ-GREEDY ALGORITHM

Initial phase:
1. $I = \emptyset$
2. Calculate $\overline{r}_{i,\infty,\infty}$ using (37)

While $\overline{r}_{i,\infty,\infty} < Th_{min}$**:**
3. $i = \arg\min_{k \in \{1,...,N\}/I} S_k$
4. Add i to I
5. Calculate $\overline{r}_{i,\infty,\infty}$ ignoring users in I, using (37)

Result:
6. $N_k = N - |I|$

TABLE III
CALCULATION OF N_k

One shall also note that this method involves virtually no computation, since the Laplace transform of Φ can be calculated numerically beforehand and tabulated, which means that the calculation of N_k simply implies looking at most N times in a table of values.

The choice of P_ϵ is critical, like in most reinforcement learning algorithms, since it quantifies how often the algorithm will lower the α despite being currently able to cover all users. The point is to try to improve the cell capacity because the current α might not be the lowest α that covers all users anymore. The problem is that by doing so users at cell edge may loose coverage. The value of P_ϵ can therefore be related to the speed at which the environment changes.

In our case we have chosen $P_\epsilon = 0.05$, that is exploring every 20 averaging periods, with an averaging period of 100ms, which means that the environment is expected to change every 2 seconds.

E. Simulation scenario

To illustrate the method described above, we have simulated it's behavior choosing Th_{min} so that users at cell edge are in the limit of coverage, namely that their mean throughput is close to Th_{min}. Simulation parameters are listed in Table IV.

Users per cell	10
Inter-site distance	$1km$
P_{max}	$16W$
Number of users	10
$t_{m+1} - t_m$	$1ms$
Averaging period	$100ms$
α	$(1, ..., 10)$
ϵ	0.05
P_ϵ	0.05
μ	3.5 (dense urban)
σ	$6.5dB$
a	0.5
b	0.5
σ_N^2	$-173dBm/Hz$

TABLE IV
MODEL PARAMETERS

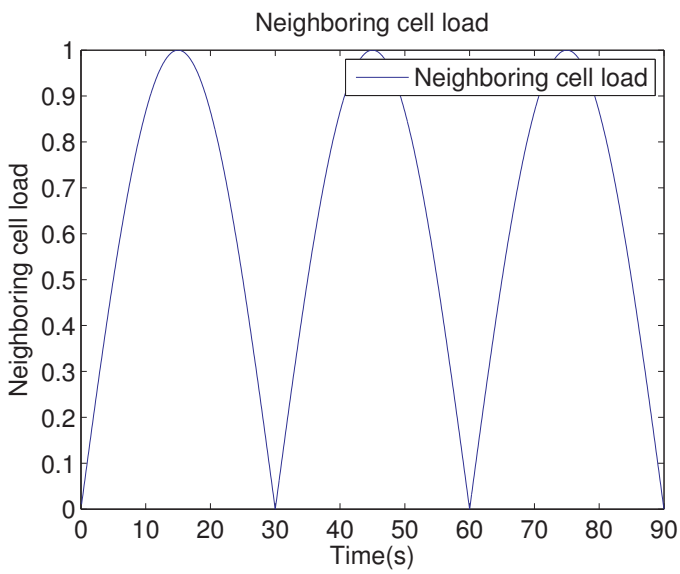

Fig. 4. Neighboring cell load (eq.(40))

F. Interpretation

Figure 4 shows the evolution of neighboring cell load during the simulation. Figure 5 shows the number of users covered by a PF policy, i.e $\alpha_t = 1 \ \forall t$, which we use as a reference, and Figure 6 shows the number of users covered using the method described in II. We can see that while the PF policy only covers approximatively half of the users during interference peaks, the self-optimization manages to cover all users almost all the time. More precisely, the cell using a PF scheduler covers on the average 5.6 users while the cell with the self-optimized PS covers on average 9.85 users which represents a significant increase in coverage.

Figure 7 shows how α evolves dynamically, and despite the relatively chaotic behavior of the network, the controller effectively follows the variations of interference. The optimization process uses available KPIs, increasing α when the interfering power increases, and decreasing it otherwise. The fact that the method effectively decreases α when the interfering power lowers is fundamental, since it guarantees that there is an optimal trade-off between capacity and coverage, since α being too high means wasting capacity. Furthermore, the method involves virtually no calculation, and it can be implemented easily in a real network.

VII. CONCLUSION

This paper has presented a self-optimization scheme based on α-fair schedulers that uses KPIs available from the network to enhance coverage and capacity. First, scheduling gains have been derived using both closed form

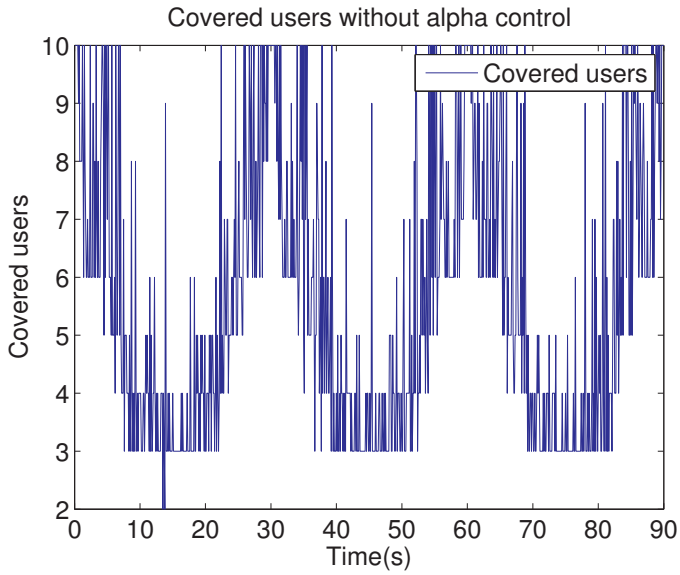

Fig. 5. Covered users under PF policy ($\alpha_t = 1\ \forall t$)

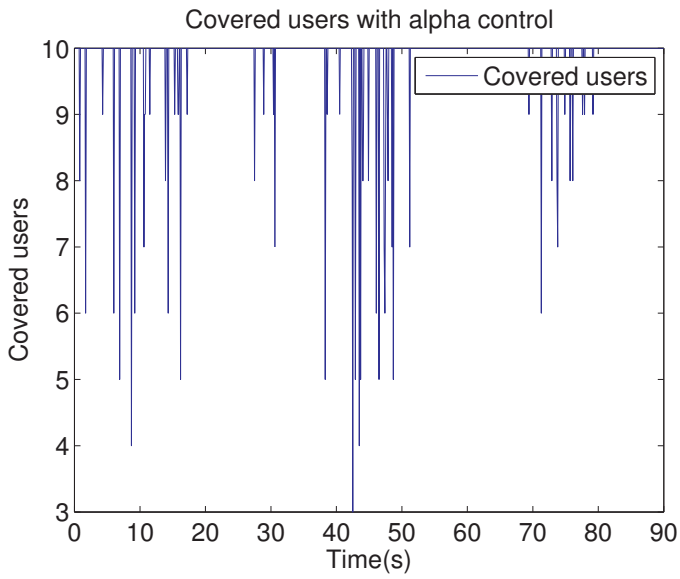

Fig. 6. Covered users with self-optimizing method (table II)

expressions and a statistical based fast algorithm. The scheduling gain computation is necessary for the design of the self-optimization scheme within a simulator. A use case of dynamic adaptation of the α-fair scheduler has been presented. Simulation results show that the self-optimization scheme considerably increases the coverage of users at cell edge and that the α-fair parameter follows the interference variation. The simplicity of the method makes it suitable for real network implementation.

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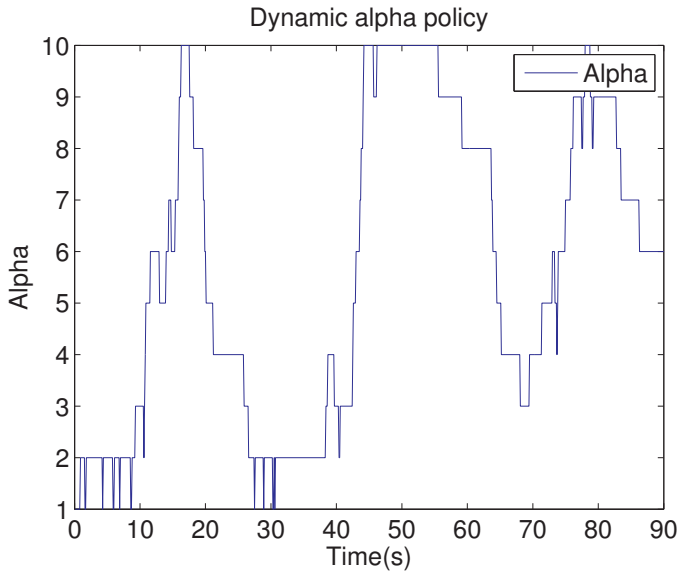

Fig. 7. Evolution of α_t with self-optimizing method (table II)

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