Cross-Layer Analysis of Scheduling Gains: Application to LMMSE receivers in Frequency-Selective Rayleigh-Fading Channels

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Outline

Introduction

The objective of this paper is to develop an Erlang-like model for flow-level evaluation of cellular networks carrying elastic traffic, taking into account:

- Drive-test results for realistic cell geometry and propagation characteristics
- Multi-user diversity gain, i.e opportunistic scheduling gain
- General receiver and channel models

Previous work

Previous work includes:

- Flow-level performance analysis for elastic traffic ([1])
- Impact of scheduling gain ([2])
- Stability of Proportional Fair (PF) scheduling ([3])
- Impact of frequency-selectivity on scheduling gain ([4])

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[1] E.Altman. "Capacity of multi-service cellular networks with transmission-rate control: a queueing analysis". In: *ACM Mobicom*. 2002, pp. 205–214.

[2] T. Bonald and A. Proutière. "Wireless Downlink Data Channels: User Performance and Cell Dimensioning". In: *ACM Mobicom*. 2003.

[3] Sem Borst. "User-level performance of channel-aware scheduling algorithms in wireless data networks". In: *IEEE/ACM Trans. Netw.* 13 (3 2005), pp. 636–647.

[4] Richard Combes, Zwi Altman, and Eitan Altman. "Scheduling gain for frequency-selective Rayleigh-fading channels with application to self-organizing packet scheduling". In: *Performance Evaluation* (2011).

Contributions of the present work

The contributions of this paper are:

- Application of Kernel Density Estimation (KDE) to fast scheduling gain calculation and convergence analysis
- Flow-level evaluation for elastic traffic and application to 3GPP models (e.g Pedestrian A 3km/h) and Linear Minimum Mean Square Error (LMMSE) receiver
- Use of network measurement results from drive tests for network dimensioning with realistic channel models.

Opportunistic scheduling: full channel statistics are needed

- Radio resources are shared in a Time Division Multiple Access (TDMA) fashion.
- The PF scheduler chooses the user maximizing the instantaneous(post-receiver) to average Signal to Interference plus Noise (SINR) ratio for transmission every scheduling instant.
- Multi-user diversity gain, compared with Round Robin(RR) scheduler
- Problem: Full channel statistics of all users are needed

$$\bar{r}_i = \int_0^{+\infty} \Phi(S_i x) \left[\prod_{j \neq i} F_j(x) \right] p_i(x) dx \tag{1}$$

Estimation of channel statistics: Kernel Density Estimation

- Full channel statistics are generally not available in closed form
- Channel statistics are estimated from samples of the instantaneous SINR using KDE ([5],[6])

$$\hat{p}_i^{(n)}(x) = \frac{1}{nh(n)} \sum_{k=1}^{n} K\left(\frac{x - X_{k,i}}{h(n)}\right) \tag{2}$$

with $h(n) > 0$ the bandwidth which decreases when n grows, and K the kernel a positive symmetrical function summing to unity.

- The channel statistics estimates give estimates scheduling gain
- Estimated scheduling gain converges to its correct value

$$\mathbb{E}[(\hat{r}_i^{(n)} - \overline{r}_i)^2] \underset{n\to\infty}{\to} 0 \tag{3}$$

[5] M. Rosenblatt. "Remarks on some nonparametric estimates of a density function". In: *Annals of Mathematical Statistics* 27 (1956), pp. 832–837.

[6] E. Parzen. "On estimation of a probability density function and mode". In: *Annals of Mathematical Statistics* 33 (1962), pp. 1065–1076.

Frequency-selective Rayleigh-fading channel

- We consider the frequency-selective Rayleigh-fading channel model
- The Pedestrian A 3km/h model is a good representation of the channel in urban dense areas ([7]).
- Channel impulse response given by Table 1.

Pedestrian A 3km/h	
Relative delay (ns)	Relative mean power (dB)
0	0
110	-9.7
190	-19.2
410	-22.8

Table: Pedestrian A 3km/h

[7]K. Larsson H. Asplund and P. Okvist. "How typical is the "Typical Urban" channel model? Mobile-based Delay Spread and Orthogonality Measurements". In: *IEEE VTC spring*. 2008.

LMMSE receiver: expression of post-receiver SINR

- LMMSE receiver, most used receiver in Beyond 3G systems
- High impact of reception techniques in wideband systems because of inter-code interference
- Post-receiver SINR calculated by channel matrix inversion ([8])

$$H = \begin{bmatrix} h_{L-1} & \cdots & h_0 & 0 & \cdots & 0 \\ 0 & h_{L-1} & \cdots & h_0 & & \vdots \\ \vdots & & \ddots & & \ddots & 0 \\ 0 & \cdots & 0 & h_{L-1} & \cdots & h_0 \end{bmatrix} \tag{4}$$

$$f = (HH^* + \frac{N_0}{E_c} I)^{-1} H_{q+L} \qquad \text{and} \qquad g = f^* H \tag{5}$$

$$SINR = \frac{|g_0|^2 \frac{E_s}{N_0}}{\frac{c}{N} \sum_{i=-q-L+1, i\neq 0}^{q} |g_i|^2 \frac{E_s}{N_0} + \sum_{i=-q}^{q} |f_i|^2} \tag{6}$$

[8] A. Saadani and J. B. Landre. "Realistic Performance of HSDPA Evolution 64-QAM in Macro-Cell Environment". In: *Vehicular Technology Conference, 2009. VTC Spring 2009. IEEE 69th*. Barcelona, 2009, pp. 1–5.

Drive-tests results

- Measurements campaign in the center of a major European town, hundreds of cells
- One typical cell selected
- Cells composed of several non contiguous parts of irregular shape, because of irregularities in propagation and cell planning.

Drive-tests results cont'd

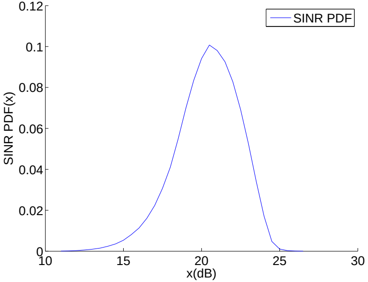

Figure: Distribution of the average SINR over the cell.

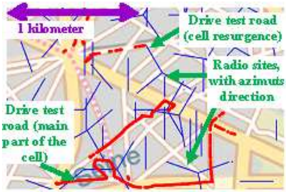

Figure: Map of the drive test for the measured cell.

Scheduling gain calculation

- The mean SINR distribution is discretized into C different radio conditions
- The state of the system is described by the number of users in each radio conditions $\mathbf{s} = (n_1, ..., n_C)$

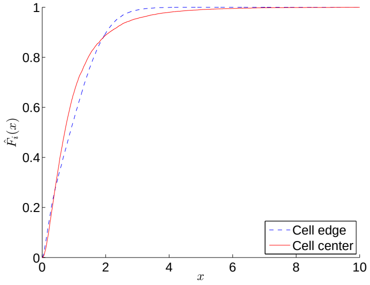

Figure: Estimation of the instantaneous SINR c.d.f for different positions in the cell.

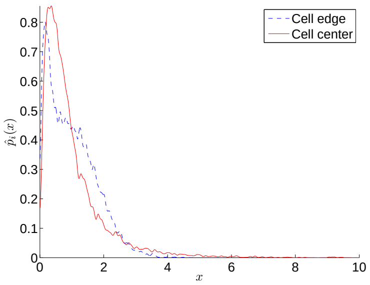

Figure: Estimation of the instantaneous SINR p.d.f for different positions in the cell.

Scheduling gain calculation cont'd

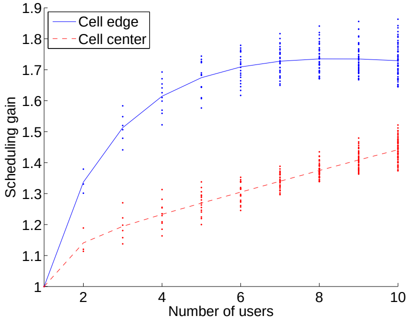

Figure: Scheduling gains at different positions in the cell.

- Channel statistics for "edge" and "center" users are very different
- The gain fluctuates around an average value, depending on the users configuration
- Good approximation for the flow level analysis: replace the gain by its trend

Advantages of the proposed method over full system simulation

- Once the channel statistics have been estimated, the scheduling gain calculation is very fast
- Three orders of magnitude faster than full system simulation
- Straightforward extension to max-throughput and max-min fair schedulers

Flow-level capacity analysis

- Users arrive at random instants to receive a file.
- Mobility is ignored, and users leave the network upon file transfer completion
- Performance metrics: stability region, file transfer time, blocking and outage rate

Markovian analysis

- Assuming Poisson arrivals and exponentially distributed file sizes, the system is Markovian ([9])
- The transition matrix is given by the previous scheduling gain calculations
- The stationary distribution is obtained by solving the set of equation below

$$\mathbf{\Pi Q} = \mathbf{0} \tag{7}$$

[9] N. Hegde and E. Altman. "Capacity of multiservice WCDMA Networks with variable GoS". In: *Wireless Networks (Springer)* 12 (2006), pp. 241–253.

Generalized Processor sharing analysis

- The scheduling gain of class c users is replaced by the average gain of class c users when there are n users in the cell $G_c(n)$
- The system is a generalized processor sharing queue ([10])
- Stationary distribution available in closed-form

$$\pi(\mathbf{s}) = \frac{1}{\Gamma} \frac{n!}{\prod_{c=1}^{C} n_c!} \prod_{c=1}^{C} \frac{\rho_c^{n_c}}{\prod_{i=1}^{n_c} G_c(i)} \tag{8}$$

[10] J. W. Cohen. "The multiple phase service network with generalized processor sharing". In: *Acta Informatica* 12 (3 1979), pp. 245–284.

Flow level capacity results

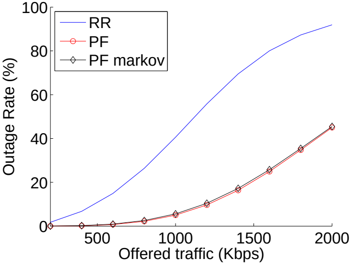

Figure: Outage rate for RR versus PF scheduling.

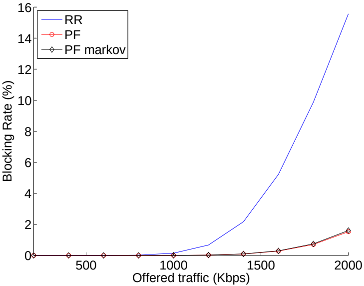

Figure: Blocking rate for RR versus PF scheduling.

Flow level capacity results cont'd

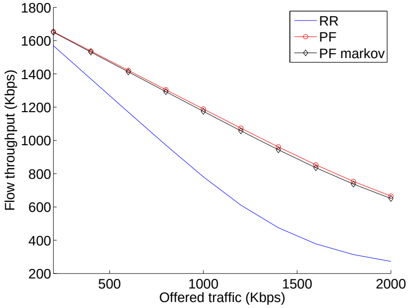

Figure: Flow throughput for RR versus PF scheduling.

- Performance metrics: outage rate, blocking rate and flow throughput
- Exact and approximated solutions are given
- The processor sharing approximation is very accurate
- For a target of 90% of users above 500 Kbps, large capacity gain of 140%.